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MegaSpin (Massive Spin search): A power tool for 21st-Century multivariate optimization

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Until recently, studying the behavior of an algorithm for minimizing loss functions over a space of considerable dimensionality (in the present instance, 100 pattern coefficients) by collecting its returns from MTRY random starts has been impractical for large MTRY except in institutions providing access to a mainframe computer. But explosive advances in computer technology now make large-MTRY Spin searches (Hyball's version being generalizable with modifications or improvements to any loss function over any multidimensional domain) feasible on an inexpensive modern desktop computer. And from seeking an effective way to make more precise the rotation-success tendencies (*a*)-(*e*) alleged in **Part I**, I have found that rather more can be learned from megaSpins than I had previously realized. Not merely can these reliably retrieve the most recoverable local optima in a criterion measure's application to a particular optimization problem, they can also make clear the differences in yield among the available parameter and procedure alternatives.

In what follows it will be expedient to say that some rotation retrieves a particular independently described pattern **B** when what this rotation has actually returned is only a pattern in the immediate vicinity (equivalently, close neighborhood) of **B**. I will not commit to an exact definition of "immediate vicinity", but Hyball operationalizes this notion under a choice of Divergence parameter GAP by taking a pattern **Q** to be in **A**'s immediate vicinity just in case, when the columns of **A** and **B** are matched for greatest similarity, no matched columns of **A** and **B** diverge by more than GAP degrees. In most of its applications here, GAP = 5.0°.

Before discussing the megaSpin results in Table 6, I should explain why search length affects these so strongly ~~these come out as they do~~. Recall that although Hyball can run a Spin series of any length MTRY, it only saves up to 99 best-by-criterion successful Trys (the Cream of this series) for similarity appraisal. (Cream limit 99 is a programming convenience that can be relaxed, but some ceiling on Cream size is needed to keep megaSpin feasible.) Suppose that $\mathcal{L}()$ is a loss function on Rot[**A**] whose optima concern us, while **B** is a pattern in Rot[**A**] whose obliquity does not exceed the run's acceptance limit.

> We presume that optimality under $\mathcal{L}$ includes satisfaction of whatever side constraints are in play, notably an obliquity limit. Hyball measures obliquity by the largest correlation between factors, and rather arbitrarily defaults its cutoff (CMAX =) to .75, which is the threshold for Try rejection used in all Spin searches reported here. Later, I will point out that CMAX can make a significant contribution to Spin search.

If **B** is among the patterns found by a Spin search, it will be counted in one of the recorded Lumps (which report the Cream patterns' recurrence frequencies) just in case its $\mathcal{L}$-rating is among the 99 best, and is also provisionally added to log store unless it differs by less than GAP from a better one already in provisional store. It follows that on any Spin series which retrieves a pattern **B** differing by less than GAP from the pattern **G** that optimizes $\mathcal{L}$, **B** or another nearly the same as **B** will be ranked first in this run's Cream. And that is true no matter how low may be the capture probability of **G**'s immediate vicinity; so even for exceptionally elusive global optima, the probability that the rank-1 pattern in a Spin series' Cream retrieves the global optimizer approaches certainty as the series length increases. In stark contrast, when **B** is at a secondary optimum of $\mathcal{L}$ each Try has an appreciable probability of getting a pattern whose $\mathcal{L}$-rating is better than any in **B**'s vicinity. In that case, increasing the series' MTRY also increases the absolute number of Tries yielding patterns both dissimilar to **B** and $\mathcal{L}$-wise superior to it, eventually leaving no room in the Spin's limited Cream for the ones close to **B**.

In short, as Spin-series length increases under fixed $\mathcal{G}$-parameters and solution procedure, its Cream tends when MTRY is small to find rather many local optima with low Lump counts and relatively low probability of returning the global best, but gives over as MTRY becomes enormous to a smaller number of larger Lumps having high probability of including patterns at the global and near-best local optima.

These effects of MTRY on Spin results are evident in Table 6, which reports Spin returns for each of 2×2 Hyball solution styles crossed with the three levels of Comp2 weighting (none; Comp2 = .8; Comp2 = .8 with KS-norming) compared in Table 5. Don't try to comprehend this at a glance; it's too cluttered for that. I'll talk about what it shows for the Comp2 alternatives under SCAN/P (Hyball's favored rotation style) with some briefer remarks about SCAN vs. STEP (unimportant for you to know) and Parallel vs. Serial (somewhat less so), and you can later judge for yourself, should you be so inclined, whether my summary of what happens under SCAN/P also holds for the other styles.

But first a word about these style contrasts: SCAN/STEP differ in how rotation of factor j within a factor plane $\langle j,k \rangle$ locates the shift in j that most improves the quality of near-zero loadings on factor k. STEP, developed in Hyball's infancy, approximates in a fast but inelegant fashion (don't ask) the criterion-optimal repositioning of j that SCAN finds by a slower but more finely discriminating search. (Now that computation time is no longer a major concern, it is unclear whether STEP has any residual benefits warranting its retention in Hyball.) Parallel/Serial, on the other hand, differ in how single-plane rotations are concatenated. Serial concatenation, which is used by Varimax, Oblimin, and so far as I know most other marketed rotation algorithms, scans all factor pairs $\langle j,k \rangle$ in some orderly sequence and unconditionally changes the factor pattern immediately after determining the optimal shift of j (and in turn k) in each current $\langle j,k \rangle$. Parallel concatenation likewise runs sequentially through all factor pairings, but collects all the recommended shift coefficients in a provisional rotation matrix that is executed only after all planes have been scanned, with the rotation's recommended movement simultaneously on all factors retarded by an adjustable shift-damping parameter. Does P vs. S matter? Table 6 has something interesting to say on this.

My strategy for choosing the MTRY levels reported in Table 6 was to start quite large (4,000 Trys, an order of magnitude larger than I had ever previously considered) and repeatedly decrease that by half until either a close match to TT was returned in Cream or further halving of MTRY seemed pointless. (In most cases, MTRY = 4,000 sufficed; but ~~fortunately~~ there are enough exceptions to exhibit the powerful effects of changing this.) The top-ranked Spin Cream reported in Table 6:*a1*, found by MTRY = 4,000 under SCAN/P with no Comp2 weighting, are quite decent approximations to TT, slipping into mediocrity on only two of the five factors; and did we not know that a much better match to target can in fact be retrieved in this case, we could be reasonably pleased by this result. Because the three leading Trys from this megaSpin are shown by their Lump counts to have comparatively high recovery probabilities, we would expect that these patterns would also have been retrieved in this Spin search's top cream had its MTRY been much smaller, say in the low 100's. And patterns Nos. 13-15 in **Part I**'s Table 5 confirm that expectation: Two of those are virtually identical (Av Div 1.3° and 2.3°) to the ones respectively ranked 3 and 2 in Table 6:*a1*. They are not, however, the return we hope for.

Tables 6:*a2*,*a3*, which exploit Comp2 weighting, tell a substantially different story. Both recover by Spin search a match to TT much closer than the best that SCAN/P can manage without Comp2 weighting – let the one in 6:*a2* (MTRY 2,000) be called R17 and the one in 6:*a3* called R1 to reflect their respective ranks though they are nearly identical (only 2.7° Av Div although they diverge on one factor by 8.0°) – but without KS-norming this superior approximation to TT is largely out of practical reach. That is because under the conditions of 6:*a2*'s Spin search many other GAP-dispersed patterns highly divergent from TT have better criterion quality than patterns near R17/R1. Lacking independent knowledge of the target pattern, our only sure way to select from Spin Cream the ones most interpretably attractive is to study these one by one (pattern appraisals availed by the Hyball-supplement programs are still problematic in how well they pick out what we should most prefer); and R17, the only close match to TT in Table 6:*a2*, is buried so deep in this Spin's Cream that we would never get to it. (A much shorter Spin search has some chance of retrieving R17 with better rank, but the odds are much against that.)

much more likely than

In happy contrast, Table 6:*a3* shows that KS-norming has promoted the patterns in R17's close vicinity (divergences from R17 of the KS-normed Trys in ranks 1,3,4 here are respectively 2.7°, 3.6°, and 5.7°) to criterion ratings little if at all inferior to global optimum with, moreover, a rather high recovery probability. And while the unwanted pattern **R5** with rank 5 in Table 6:*a3*'s Cream is ~~thrice as likely as~~ R1 to be found by any one Try, its inferior criterion rating insures that **R5** will never dislodge R1 from 1st rank, much less squeeze it out of the Cream altogether with increasingly large MTRY, so long as MTRY is large enough to make retrieval of both R1 and **R5** likely. (Patterns No. 23,24 in Table 5 nicely confirm this expectation.) So Comp2 weighting supplemented by KS-norming has in this instance made it nearly certain that a Spin search of decent length will return **TT** as rank 1 in its Cream. (Note that this is true not merely of Hyball's current implementation of SCAN/P but of its other style variants as well.)

I'll overview the remainder of Table 6 more briefly.

STEP style. Apart from STEP/P's incompetence when it tackles the present job without Comp2 weighting (for much the same reason too digressive to explain, STEP and Oblimin are both especially vulnerable to complexity-2 items), Tables 6:b show the same pattern of TT retrieval under varied NTRY as does SCAN/P: Under Comp2 = .8, STEP/P too returns a very nice match to **TT** which however is buried behind poor matches that rank higher than the good one in criterion quality unless KS-norming is also invoked. In that case, STEP/P too gets an excellent match of **TT** in rank 1 of almost every Spin search of reasonable length. And – an unrelated point – the five length-varied Spin series under STEP/P without Comp2 weighting also show, as do likewise their Serial-style counterparts, how the abundance of low-count Lumps generally found when MTRY is on the order of 100 gives way with increasing long search to a small number of high-count lumps around patterns at the criterion measure's global and leading-secondary optima.

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Iteration style. The Serial vs. Parallel contrast here is especially instructive. Each Serial Spin search yields results quite similar to its Parallel counterpart, including the quality of results obtained, with one striking exception: Serial rotation consistently returns fewer Lumps from Spin search of a given length than does Parallel rotation. (It also has faster execution time, which no longer matters much.) What this tells us is that Serial rotation is less sensitive than Parallel to the criterion measure's surface texture; it overlooks some local optima that the other detects. That is no loss in the present application, since the best Serial approximations to **TT** are as good as their Parallel counterparts. But that may not always be the case. Lacking evidence or argument to the contrary, we must expect that some patterns in Rot[A] which might especially interest us were we to find them can be retrieved by Parallel but not Serial Spin search. In contrast, it appears (though the evidence is still meager) than any pattern recoverable by Serial Spin is also likely to appear in the Cream of its Parallel counterpart. This strengthens my past judgment that in Hyball, SCAN/P should be the rotation style of choice. Whether it also has import for the programming of other iterated optimizations I am in no position to say.

Oblimin parameters. Finally, Table 7 reports megaSpin findings on Oblimin that may surprise you. Just as Varimax and Equimax are parameter variants within the Orthomax family, what I have been calling "Oblimin" is just the default instance of a family whose parameter, Gamma, can be any real number albeit positive values range in benefit from unhelpful to degenerate. In a previous simulation study I had observed that although Gamma = 0 (Oblimin's default) seemed generally so close to best that opting for an alternative would seldom be worth the dither, there were hints that a small single-digit negative Gamma may sometimes recover complexly simple-structured targets slightly better than default Gamma. So when running a few megaSpins to confirm Oblimin's apparent insensitivity to start position, it is of interest to see whether this can also clarify what Gamma does for Oblimin.

Actually, Table 7 reports few megaSpin results for Oblimin(0), because when MTRY was multiplied beyond 200 the returned Cream generally contained just one or two patterns, with the Rank-1 Try enormously dominant in Lump count and any following in higher rank closely matching this leader. Even so, small MTRY reveals a spread of returns here worth noting. In Table 7*a*'s results for relatively easy target T (Table 2*a*), almost all the Trys returned as Cream closely resemble one another both within and between Gamma alternatives, with about the same good-but-not-great match to T. But each Gamma setting also managed to pick up an outlying clinker. And observe the difference in lumping: Even under small MTRY, which maximizes the diversity of patterns retained in Cream, Oblimin(0) returns just one or two stragglers outside of one massive Lump, whereas Oblimin(-1) speads its returns over several local optima.

And the plot thickens when Oblimin is unleashed to pursue tough target TT. Note first of all that the approximations to TT found in Table *7b* under Oblimin(-1), though poor matches, are nevertheless considerably more accurate than Oblimin(0)'s all-worthless returns. And surprisingly (though I have no idea whether there is anything to make of this), apart from a couple of clinkers these all approximate TT to nearly the same degree of overall accuracy. But the most overwhelming difference in how the two tested Gammas respond to complex pattern TT is in the bumpiness of optimality surface from which they return local optima. Just as in Table *7a*, the Oblimin(0) Trys on which Table *7b* reports almost always finish so close to global optimum that only when GAP remains quite small (5°) do these segregate into Lumps that are distinguishable even if highly congruent. (Whether these minimally divergent lumps contain genuinely distinct local optima of the loss-measure defined by Oblimin(0) or only assorted imperfect convergences to the very same local optimum I do not know. I suspect the latter to at least some extent, but for simplicity will ignore it.) In contrast, the Lumps returned by Oblimin(-1) under GAP = 5° were so abundant even for MTRY in the thousands that it would bewilder to show them. Seeking a more perspicuous grain of resolution, Table *7b* concludes with the Lumps found when vicinities are coarsened by successive GAP increments of 5°. Even under huge search length (MTRY = 8,000), the distinguished Lumps don't condense into a small count until GAP is expanded to 20°.

There is no need for me to dwell on the finer features of Table *7b*; the data are there for you to take from them what you will. But two overview conclusions have some importance, one for users of Oblimin and the other for students of nonlinear optimization. Regarding Oblimin, its Gamma settings 0 vs. -1 needn't make much difference for the rotated patterns it returns when target contains a strong contingent of complexity-1 items (cf. Table 1*a*). But even in that case Oblimin(-1) is more sensitive to start position than is Oblimin(0) despite there not being much divergence among most of these start-influenced returns; and for more complex targets there may be little resemblance between the nearly-start-invariant returns from Oblimin(0) and the vast dispersion of start-sensitive returns from Oblimin(-1) which, if the present instance is typical, are all much closer to target than is Oblimin(0)'s output.

But that matters only for Oblimin enthusiasts, whom I would prefer to sell on the superiority of Hyball's native rotation styles when target complexity gets tough. Rather less parochial is the lesson to take from the megaSpin prowess demonstrated here. Study of the Spin Cream returned under a diversity of MTRY levels and GAP settings for a particular loss-function $\mathcal{L}()$ in a particular application (this could be any high-dimensional optimization, not just factor rotation) reveals surprisingly much about the contours of $\mathcal{L}$'s response surface in this application as well as how a particular style of solving for $\mathcal{L}$'s optima compares to other styles. (How easily we can separate style effects from $\mathcal{L}$-contours I don't know.) In Table *7b* we apparently see that the response surface defined by Oblimin(0) over Rot[TT] has an enormous sinkhole with smooth walls, narrow bottom, and mouth wide enough to catch almost everything thrown at it. In large contrast, Oblimin(-1)'s response surface, has two main concavities, not far apart, with broad catchments having roughly equal area and mildly sloping sides sufficiently pitted to trap much of what comes their way. How best to exploit such information about an optimization procedure remains unclear; but when lazars were first created they too were a laboratory curiosity looking for an application. (No, I don't think that megaSpin is SMEP's answer to the lazar; but a guy can wish.)

TABLE 6. Studying a criterion measure's local optima by Spin search

Matches of target pattern TT to Spin Cream under assorted Hyball rotation styles. Patterns saved from each Spin search here are the 99 ordered best-by-criterion out of MTRY Tries (initially MTRY = 4,000 for each style examined), followed by filtering at GAP = 5.0 (no matched factors diverging by more than 5°). In each strip-table below, the parenthesized Lump count below the pattern ranked j = 1,2,... is the number of patterns filtered out by proximity to retained pattern j, plus 1 to include the one retained. (Lump thus shows the number of patterns found in the vicinity of each local optimum in order of decreasing optimality.)

V flags TT's best match in each Spin series

COMP2 WEIGHTING UNDER PARALLEL-ITERATED ROTATION

a1) SCAN/P, Comp2 = nil; MTRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	V																					
TT	9.5	10.8	13.2	15.8	10.5	16.4	27.0	24.7	23.4	31.4	22.6	25.7	17.2	22.9	22.1	23.1	22.5	20.7	35.5	19.0	22.4	31.3
Lump	(7)	(11)	(17)	(2)	(1)	(2)	(7)	(1)	(1)	(2)	(3)	(1)	(3)	(3)	(1)	(16)	(1)	(4)	(4)	(7)	(4)	(1)

a2) SCAN/P, Comp2 = .8, no KS-norming; MTRY = 4,000, 2,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
											V			
TT	35.1	37.2	39.4	37.0	39.9	37.0	41.2	36.7	41.6	41.2	22.1	39.3	38.8	45.3
Lump	(7)	(1)	(15)	(10)	(1)	(1)	(1)	(2)	(3)	(2)	(53)	(1)	(1)	(1)

Rank in Cream of 2,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
																	V			
TT	35.1	36.7	38.5	37.7	41.9	41.5	36.7	36.9	21.8	39.2	36.9	35.5	41.9	22.5	36.2	38.1	7.5	15.3	32.9	19.8
Lump	(4)	(6)	(3)	(1)	(1)	(1)	(1)	(3)	(45)	(1)	(1)	(1)	(1)	(2)	(1)	(2)	(13)	(1)	(10)	(1)

a3) SCAN/P, Comp2 = .8 with KS-norming; MTRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13
	V												
TT	6.6	14.4	6.7	7.0	22.2	29.0	29.3	14.0	12.1	11.5	31.8	22.7	29.1
Lump	(17)	(4)	(1)	(2)	(56)	(5)	(4)	(1)	(2)	(1)	(4)	(1)	(1)

b1) STEP/P, Comp2 = nil; MTRY = 4,000, 2,000, 1,000, 500, 250

No match to TT closer than Div = 28°, which occurred in rank 10 of the shortest search. The 4,000-Try search returned just three lumps, the next three longer search lengths each found four lumps, and the 250-Try got 20. All but one return from each of the four longest searches as well as rank 1 in the shortest closely matched a common pattern whose divergence from TT was about 35° and whose recurrence frequency was higher – in all but the shortest search overwhelmingly higher – than all other recovered patterns combined.

b2) STEP/P, Comp2 = .8, no KS-norming; MTRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
							V								
TT	36.0	36.7	22.9	36.5	22.1	22.0	6.6	34.6	7.8	32.0	35.3	31.8	13.6	8.2	35.0
Lump	(1)	(1)	(25)	(35)	(2)	(3)	(5)	(5)	(8)	(1)	(9)	(1)	(1)	(1)	(1)

b3) STEP/P, Comp2 = .8 with KS-norming; MTRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	V															
TT	5.8	7.2	14.4	22.5	29.0	10.4	13.8	23.1	11.4	15.5	8.0	36.9	12.4	24.6	23.2	34.2
Lump	(3)	(1)	(1)	(27)	(5)	(1)	(6)	(3)	(1)	(1)	(1)	(36)	(6)	(1)	(4)	(1)

GRADES OF COMP2 WEIGHTING UNDER SERIAL-ITERATED ROTATION

c1) SCAN/S, Comp2 = nil; MIRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7	8	9	10	11
TT	9.6	13.6	11.3	16.1	22.4	21.0	23.3	24.3	36.3	22.7	36.2
Lump	(12)	(13)	(7)	(4)	(28)	(6)	(8)	(8)	(8)	(4)	(1)

c2) SCAN/S, Comp2 = .8, no KS-norming; MIRY = 4,000, 2,000, 1,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7
TT	36.7	38.9	40.2	41.0	35.9	41.4	19.6
Lump	(50)	(32)	(2)	(1)	(11)	(2)	(1)

Rank in Cream of 2,000 Trys

	1	2	3	4	5	6	7
TT	37.0	38.7	36.2	41.2	21.9	36.4	33.2
Lump	(17)	(13)	(8)	(5)	(54)	(1)	(1)

Rank in Cream of 1,000 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13
TT	36.9	38.9	35.8	41.2	21.9	7.0	36.5	14.3	33.0	35.9	23.7	41.0	26.6
Lump	(7)	(9)	(3)	(7)	(32)	(10)	(3)	(4)	(20)	(1)	(1)	(1)	(1)

c3) SCAN/S, Comp2 = .8 with KS-norming; MIRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5
TT	6.5	14.6	29.3	22.2	43.7
Lump	(25)	(6)	(4)	(63)	(1)

d1) STEP/S, Comp2 = nil; MIRY = 4,000, 2,000, 1,000, 500, 250

Rank in 4,000 Trys

	1	2	3
TT	36.0	31.4	40.7
Lump	(91)	(7)	(1)

In 2,000

	1	2
TT	35.7	31.6
Lump	(92)	(7)

In 1,000

	1	2	3
TT	35.7	31.4	41.0
Lump	(93)	(5)	(1)

In 500

	1	2
TT	35.7	31.5
Lump	(96)	(3)

Rank in 250 Trys

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TT	35.8	31.4	22.5	37.6	24.4	29.5	28.3	25.4	35.8	29.4	31.4	41.1	28.2	40.7
Lump	(51)	(3)	(1)	(3)	(1)	(5)	(3)	(1)	(14)	(3)	(2)	(8)	(1)	(3)

d2) STEP/S, Comp2 = .8, no KS-norming; MIRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4
TT	22.7	36.6	6.4	32.7
Lump	(54)	(40)	(4)	(1)

d3) STEP/S, Comp2 = .8 with KS-norming; MIRY = 4,000

Rank in Cream of 4,000 Trys

	1	2	3	4	5	6	7
TT	6.4	29.6	14.6	22.7	14.3	10.8	43.9
Lump	(15)	(16)	(3)	(56)	(7)	(1)	(1)

Table 7. Behavior of Oblimin under two choices of its Gamma parameter in Spin search for easy vs. hard simple-structured targets. Each tabled entry is mean Divergence over the marginally identified Trys' five matched columns.

a) Recovery of Table 2's easy target T (No. 1 in logfile). All results for this target were collected on a single Hyball run, which is why the pattern Nos. run consecutively over all Trys from four Spin Creams, two each under Gamma = 0 and Gamma = -1.

MIRY:	200			100				200									100					
Gamma:	0			0				-1									-1					
Rank:	1	2	1	2	3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No. :	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
T	14.3	15.3	14.6	14.9	28.6	13.2	14.7	15.6	14.2	14.4	15.1	14.2	13.6	14.3	14.1	15.6	14.6	15.7	15.4	18.8	18.5	33.8
Lump	(98)	(1)	(97)	(3)	(1)	(35)	(29)	(14)	(15)	(3)	(3)	(15)	(12)	(19)	(15)	(13)	(14)	(5)	(3)	(1)	(1)	(1)

No.	2	3	4	5	6
2	.0				
3	2.9	.0			
4	1.2	1.9	.0		
5	2.3	4.2	3.1	.0	
6	20.0	19.8	19.5	21.4	.0

No.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
2	2.7	4.3	4.5	5.1	4.3	4.8	3.2	4.3	3.5	4.5	5.1	5.5	5.7	5.2	12.0	21.2	26.2
3	4.1	6.3	4.0	4.3	6.4	5.5	3.3	4.0	5.9	5.4	4.0	3.8	5.2	7.2	14.1	21.4	25.5
4	2.8	5.0	4.0	4.4	4.9	4.6	2.8	3.7	4.4	4.4	4.4	4.6	5.1	5.8	12.9	21.1	26.2
5	4.1	4.2	5.1	6.7	5.2	6.2	4.1	5.9	3.7	6.1	6.2	7.0	7.2	5.4	10.2	22.7	25.5
6	19.3	22.7	20.3	18.9	20.5	20.4	19.9	18.3	22.6	19.7	19.5	19.6	17.4	22.3	24.5	27.0	12.0

No.	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
7	.0																
8	3.8	.0															
9	3.9	4.0	.0														
10	4.2	7.0	4.4	.0													
11	3.5	2.8	5.2	6.2	.0												
12	4.1	4.4	4.4	4.6	3.4	.0											
13	2.2	3.9	2.4	4.1	4.8	4.5	.0										
14	3.0	6.2	4.5	3.0	4.8	4.6	3.4	.0									
15	3.0	1.9	4.4	6.5	2.1	4.5	3.9	5.3	.0								
16	3.5	4.6	4.5	4.3	3.2	1.6	4.4	4.0	4.3	.0							
17	4.5	6.1	2.9	2.3	6.0	4.3	3.6	4.2	6.0	4.3	.0						
18	5.1	8.1	5.0	2.1	7.5	5.9	4.5	3.6	7.5	5.8	3.4	.0					
19	5.3	7.5	5.1	3.7	6.1	4.8	5.4	4.4	6.9	4.2	3.2	4.9	.0				
20	4.8	2.7	5.3	7.6	2.9	3.8	5.3	6.9	2.7	4.6	6.6	8.7	7.6	.0			
21	12.5	10.7	13.1	15.7	11.9	14.4	13.0	14.3	10.4	14.2	15.3	16.3	16.0	11.6	.0		
22	20.0	22.8	24.4	19.9	21.1	21.5	21.3	19.2	21.8	20.5	21.8	19.8	21.3	23.3	27.5	.0	
23	26.3	26.2	25.7	25.7	27.1	27.0	26.3	26.9	26.4	26.7	24.9	26.0	22.0	27.2	28.3	34.4	.0

b) Recovery of Table 3's tough target TT (No. 1 in logfile). The returns shown here exactly parallel those in subtable 7a except for differences in the grain of Spin search (MIRYs and GAP) to accomodate the strong effect of increased target difficulty on Oblimin(-1)'s response and a minor change in display layout reflecting that results below are taken from several different Hyball runs.

SPIN CREAM FOR OBLIMIN Gamma = 0 (small GAP)

GAP :	5.0°		5.0°		5.0°		5.0°			
MIRY:	1,000		5000		250		125			
Rank:	1	2	1	2	1	2	1	2	3	4
No. :	2	3	4	5	6	7	8	9	10	11
	V		V		V				V	
TT	33.9	35.9	33.3	35.1	33.1	34.3	33.9	35.6	33.7	37.4
Lump	(98)	(1)	(93)	(6)	(88)	(11)	(75)	(17)	(6)	(1)

No. :	2	3	4	5	6	7	8	0	10	11
2	.0									
3	4.1	.0								
4	1.1	4.8	.0							
5	2.6	3.1	3.4	.0						
6	1.6	5.1	.6	3.7	.0					
7	1.7	3.1	2.3	1.4	2.7	.0				
8	.7	3.9	1.5	2.0	1.8	1.2	.0			
9	3.1	2.4	3.9	.9	4.2	1.9	2.6	.0		
10	2.3	3.6	2.5	3.3	2.7	2.2	2.5	3.6	.0	
11	24.5	23.0	25.1	23.2	25.3	23.9	24.2	22.9	25.0	.0

SPIN CREAM FOR OBLIMIN Gamma = -1 (medium to wide GAPs)

[GAP = 5° results omitted; returns overly profuse even from megaSpins. Tables of divergences among]
[the Trys within and between the Spin Creams variously under MIRY = 200,100,2,000,8,000 are omitted]
[beyond one excerpt because these only show at great expanse the same finely pebbled texture of]
[divergences, ranging up to 32° at MIRY = 8,000 and higher at lower MIRY, that is mostly except to]
[demonstrate the enormous diversity of patterns returned in Spin Cream under Oblimin(-1) at all]
[search lengths.]

MIRY:	200											
GAP:	10°											
Rank:	1	2	3	4	5	6	7	8	9	10	11	12
No. :	2	3	4	5	6	7	8	9	10	11	12	13
		V										
TT	22.2	22.0	26.9	24.8	25.9	23.8	24.1	23.5	24.2	23.7	27.5	34.0
Lump	(5)	(1)	(3)	(10)	(39)	(2)	(8)	(3)	(2)	(5)	(20)	(1)

MIRY: 100 (continuing the above Hyball run with decreased MIRY)

GAP:	10°														
Rank:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. :	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
						V									
TT	23.1	24.2	23.6	25.7	23.3	21.4	24.0	24.0	26.8	25.1	23.7	26.0	25.5	24.5	30.0
Lump	(14)	(9)	(5)	(6)	(1)	(1)	(34)	(1)	(1)	(8)	(2)	(12)	(1)	(3)	(1)

MIRY: 2,000 (a fresh Hyball run)

GAP:	15°								20°			
Rank:	1	2	3	4	5	6	7	8	1	2	3	4
No:	2	3	4	5	6	7	8	9	10	11	12	13
TT	24.2	22.8	24.5	24.7	25.3	26.7	23.8	48.7	21.9	23.6	25.2	27.9
Lump	(6)	(2)	(35)	(1)	(3)	(50)	(1)	(1)	(18)	(3)	(52)	(26)

MIRY: 8,000 (another fresh Hyball run)

GAP :	10°																
Rank:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No. :	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
TT	22.3	22.4	24.5	23.0	23.0	26.7	23.0	23.6	23.7	25.5	24.2	26.9	22.3	22.8	28.1	24.0	28.3
Lump	(18)	(1)	(16)	(7)	(1)	(4)	(2)	(2)	(2)	(1)	(25)	(15)	(1)	(1)	(1)	(1)	(1)

MIRY: 8,000 (continuing the above Hyball run with wider GAP neighborhoods)

GAP :	15°									20°				
Rank:	1	2	3	4	5	6	7	8	9	1	2	3	4	5
No. :	19	20	21	22	23	24	25	26	27	28	29	30	31	32
TT	22.9	25.6	23.5	23.4	25.3	23.6	25.6	28.8	34.5	23.0	27.0	23.4	28.2	37.3
Lump	(34)	(7)	(3)	(1)	(47)	(2)	(3)	(1)	(1)	(39)	(3)	(51)	(5)	(1)

MIRY: 8,000 (excerpting the double-digit Lumps from this run's Try-divergence table)

GAP :	10°				15°		20°	
Rank:	1	3	11	12	1	5	1	3
No. :	2	4	12	13	19	23	28	30
2	.0							
4	10.2	.0						
12	14.1	10.5	.0					
13	18.1	15.2	5.9	.0				
19	5.3	12.7	11.5	14.6	.0			
23	16.1	12.3	3.6	3.6	13.3	.0		
28	3.6	8.6	13.0	17.6	5.5	15.2	.0	
30	14.1	6.8	6.9	10.7	15.5	7.6	12.7	.0