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Studies in the Empiricist Theory of Scientific Meaning

Part II – On the Equivalence of Scientific Theories

Abstract

Drawing upon the Carnapian explication of "analytic truth," Part II examines a possible axiomatic basis for the empiricist theory of scientific meaningfulness to demonstrate that even if theoretical terms are able to designate entities inaccessible to the observation language, as held by Empirical Realism, so long as the meanings of theoretical terms derive from their connections with the observation language, the meaning content of a theory is exhausted by its observational consequences.

When are two scientific theories equivalent in what they say about the world? The empiricist is strongly inclined to answer, "When they have the same observational consequences." To the extent he would also like to give theoretical concepts a realistic interpretation, however, the empiricist must not leap too hastily at such a conclusion, for if theories are able to make assertions about entities inaccessible to the observation language, as maintained by Empirical Realism, it must seriously be considered whether two theories might not differ in their total factual commitments even though the observation-language subsets of these commitments are in agreement. Nonetheless, despite this need for wariness, the empiricist's intuition is a sound one: Even if theoretical terms have extra-observational reference, if such terms acquire their meanings through their connections, via the theory in which they are embedded, with the observation language, powerful arguments can be developed to show that theories which are observationally equivalent must be equivalent in meaning as well. One such argument, based on a conceptual approach due to Carnap, will be presented here.

Apart from heuristic asides, no attempt will be made to bring philosophical clarity to the semantical concepts, such as "meaning," "analytic truth," etc., here formalized. The purpose of the present analysis is to suggest a possible axiomatic basis for part of the empiricist theory of scientific meaningfulness and to draw certain extremely important logical consequences of these axioms. These deductive relationships hold irrespective of whatever ultimate analysis is given to the semantical content of the system; in fact, such a formalization substantially assists insight into the rather treacherous complications which arise in attempts to explicate the meanings of scientific theories.[1] As for justifying this particular choice

[1]For an extensive philosophical exploration of these problems, see Rozeboom, 1962.

of axioms, it should suffice here to note that they are either (a) straightforward applications of the Carnapian explication of A-truth (Assumptions 1 & 3), (b) semantical tenets which are widely accepted and, in the present context, non-controversial (Assumptions 2, 4 & 7), or (c) apprently necessary consequences of the empiricist interpretation of theoretical meanings (Assumptions 5, 6 & 8).

The proofs to follow turn on the formal validity of certain formulas occurring in the system of languages here scrutinized, where "formally valid" means, as usual, "true in any model of the language." Several definitions of "model" have appeared in the technical literature, almost any of which would suffice here; however, since the present analysis presupposes a Carnapian framework, we may as well, to be explicit, adopt Carnap's[2] usage, which is essentially the standard model of Gödel. Those theorems about formal validity needed here, namely, some results from the propositional calculus and a few elementary principles of quantification theory, are sufficiently familiar that they will be presupposed without explicit mention.

As is customary, we shall here make use of structural notation for reference to expressions in the language under discussion. Thus if '$A(\phi)$' and 'S' designate, respectively, a (possibly complex) predicate and sentence in L, '$S \cdot (\exists\phi)A(\phi)$' designates the sentence in L formed by conjunction of S with the existential quantification of $A(\phi)$. (More precisely, 'S' '$A(\phi)$', '$S \cdot (\exists\phi)A(\phi)$' etc., are metalinguistic variables which range over expressions of the indicated logical forms in L.) In a few instances, we shall also introduce, in brackets, further simplifying abbreviations, notably, 'T' for '$T(\tau_1, \ldots, \tau_n)$'.

Let L_o be an observation language with the usual syntactic properties, including the formation of existence-statements by $\exists$-quantification over any primitive descriptive constant. As defined here, L_o corresponds to, and, for explicitness, may be identified with, Carnap's "logically extended observation language" (Carnap, 1963). By an "accepted theory," let us mean an expression $T(\tau_1, \ldots, \tau_n)[T]$ such that: (*a*) $T(\tau_1, \ldots, \tau_n)$ is the conjunction of all postulates containing theoretical terms which have been independently accepted.[3] ("Independently accepted" is here meant to indicate that theoretical sentences which are analytic consequences of T need not themselves be included in T.) (*b*) There are variables $\phi_1, \ldots, \phi_n$ such that $T(\phi_1, \ldots, \phi_n)$ is a (presumably complex) predicate in L_o. (*c*) $\tau_1, \ldots, \tau_n$ are distinct terms which are not terms in L_o, but which are manipulated syntactically as are primitive descriptive terms of corresponding logical types in L_o. Then according to the empiricist, the τ_i acquire meaning—within limits, the precise de-

[2]See Carnap, 1958, p. 173.

[3]Strictly speaking, "acceptance" of a set of postulates is relative to a particular theory-user at a particular time. Explicit reference to person and time is here unnecessary, however, for differences in meanings resulting from differences in the theories accepted by various persons at various times are subsumed under differences within the family of languages $L_{(T_i)}$ examined below.

termination of which is still a major pursuit of logical empiricism—by their use with L_o in this way.

More generally, we shall mean by "theory T," accepted or otherwise, the statement that T *would* be *were* it to comprise the totality of independently accepted theoretical postulates. Then while the *sign-design* T may have various meanings, or none, according to whatever accepted theory is imparting meaning to the theoretical terms, the meaning of *theory* T is the meaning that the sign-design T has in the language (see below) in which it is the accepted theory.

The meaning content, or "force," of a statement, observational or theoretical, is in some sense the set of beliefs to which a person who accepts that statement is committed. The meaning content of one statement S_2 is included in that of another, S_1, if and only if acceptance of S_1 includes commitment to whatever beliefs are expressed or entailed by S_2. But this also seems to be what we have in mind when we say that S_1 "analytically implies" S_2, or that $S_1 \supset S_2$ is "analytically true" [A-true]. Hence *within* a given language, we may explore relations of meaning content in terms of A-truth—e.g., S_1 and S_2 are equivalent in meaning[4] in language L if and only if $S_1 \equiv S_2$ is A-true in L. However, meaning equivalence is broader than analytic equivalence, for "A-truth" is an intra-linguistic concept only, whereas meaning relations transcend the boundaries of a given language. For example, 'Snow is white' (in English) is equivalent in meaning to 'Schnee ist weiss' (in German), yet 'Snow is white if and only if Schnee ist weiss' is not an A-truth of any language. As will become clearer as we go along, this poses a special problem for analyzing the meaning contents of theories. Nonetheless, the concept of "A-truth" will be of assistance if we assume, with Carnap, that it can be defined in terms of formal validity and a set of "meaning postulates" (Carnap, 1952, 1963).

Preliminary Assumption. There is a set of sentences in L_o whose conjunction, A_o, is such that a sentence S is A-true in L_o if and only if $\vdash A_o \supset S$ in L_o (i.e., $A_o \supset S$ is formally valid in L_o). The sign '$\vdash$' is here taken to signify formal validity, rather than provability, since it may well be that not all formally valid sentences in L_o are provable.

Let $L_{(T)}$ be the enriched language formed from L_o by addition of the terms $\tau_1, \ldots, \tau_n$ which are introduced when the theory $T(\tau_1, \ldots, \tau_n)$ is accepted. (Sentences in L_o are therefore a subset of sentences in $L_{(T)}$, and a proper subset if $n > 0$. L_o itself is to be understood as the special case of $L_{(T)}$ in which T is the null-theory—i.e., L_o is the language in which no theoretical postulates have been accepted.) What can we say about A-truth in $L_{(T)}$? If acceptance of T adds any

[4] As used here, "meaning equivalence" does not necessarily imply *identity of meaning*, but only mutual entailment by virtue, if necessary, of the meanings involved. For example, two tautologies entail each other and are hence equivalent in meaning in the present sense, but 'John = John' and 'Peter is tall $\equiv$ Peter is tall' are certainly not *identical* in meaning.

new meaning postulates, A_T, to the language, the definition of "theory" implies that $T \supset A_T$ is A-true. But A-true with respect to what set of meaning postulates, A_o or $A_o \cdot A_T$? A little reflection shows that unless $\vdash T \cdot A_o \supset A_T$ in $L_{(T)}$ addition of A_T as a meaning postulate goes beyond the mere acceptance of T, and would amount, in effect, to replacing T with the enriched theory $T \cdot A_T$. Hence,

Assumption 1. A sentence S is A-true in language $L_{(T)}$ only if $\vdash A_o \cdot T \supset S$ in $L_{(T)}$.

A sentence is inconsistent if and only if its acceptance commits one both to believe and to disbelieve some proposition, which, by *modus tollens* and the Law of Contradiction, is the same as saying that the sentence analytically implies its own denial. But $S \supset \sim S$ is A-true in $L_{(T)}$ if and only if $\sim S$ is also A-true in $L_{(T)}$. Hence,

Assumption 2. A sentence S is inconsistent in language $L_{(T)}$ if and only if $\sim S$ is A-true in $L_{(T)}$. It follows from Assumptions 1 and 2 that S is inconsistent in $L_{(T)}$ only if $\vdash A_o \cdot T \supset \sim S$ in $L_{(T)}$ For the special case where T is the null-theory, S is inconsistent in L_o only if $\vdash A_o \supset \sim S$ in L_o.

Assumptions 1 and 2 provide necessary but not sufficient conditions for A-truth and inconsistency in $L_{(T)}$ While exhaustive determination of sufficient conditions depends upon identification of those meaning postulates, if any, introduced by acceptance of T, certainly one condition which suffices is

Assumption 3. A sentence S is A-true in language $L_{(T)}$ if $\vdash A_o \supset S$ in $L_{(T)}$. The intuitive grounds for this Assumption, which would be a theorem in a more complete axiomatization of "A-truth," are made more explicit in Assumption 6, below. It will be noted that the Preliminary Assumption follows from Assumptions 1 and 3 by taking T to be the null-theory.

Under what conditions are two sentences incompatible? Intuitively, S_1 and S_2 are incompatible if and only if acceptance of S_1 commits one to deny what S_2 asserts, and conversely. If S_1 and S_2 belong to the same language, this is equivalent to saying that the conjunction of S_1 and S_2 is inconsistent. Hence,

Assumption 4. Two sentences S_1 and S_2 are incompatible in language $L_{(T)}$ if and only if $S_1 \cdot S_2$ is inconsistent in $L_{(T)}$.

A problem in inter-linguistic comparisons arises, however, when we attempt to formalize the conditions under which two theories are incompatible. Theory T_1 is incompatible with theory T_2 if and only if the belief commitments which would be made were T_1 accepted are incompatible with those which would be made were T_2 accepted. But joint acceptance of the postulates of T_1 and T_2 is acceptance of a *third* theory, $T_1 \cdot T_2$, the belief commitments of which may go beyond any made by either T_1 or T_2. That is, if T_1 is an accepted *theory*, not just one of the postulates in a more inclusive theory, the sentence T_2 will, in general, not be in the language $L_{(T)}$ at all, and even if it is (as will occur if T_1 and T_2 use the same sign-designs for their theoretical terms), the meaning content of T_2 in $L_{(T_1)}$ will be different from its meaning content in $L_{(T_2)}$ if acceptance of T_1 gives the theoretical terms different meanings than would be given them by acceptance of T_2. Hence we cannot judge the incompatibility of theories T_1 and T_2 merely by examining the combined theory $T_1 \cdot T_2$ for inconsistency. On the other hand, it seems intuitively clear that when a sentence containing theoretical terms is a postulate of a more inclusive theory, it must there have at least the meaning content it would have were it accepted as a theory by itself, since addition of new postulates to a theory can surely not *weaken* the linguistic force of the old postulates. Thus a theory $T_1 \cdot T_2$, formed by conjoining the postulates of theories T_1 and T_2, must have at least the belief commitments of both T_1 and T_2. Hence,

Assumption 5. Two theories T_1 and T_2 are incompatible only if the combined theory, $T_1 \cdot T_2$, is inconsistent in $L_{(T_1 \cdot T_2)}$.

Of course, Assumption 5 (or Assumption 3, for that matter) would not be reasonable if the meanings of observational terms did not remain constant from one $L_{(T_i)}$ to another. In fact, the assumption that expressions in L_o are unaltered in meaning by acceptance of a theory is so basic to the empiricist's program of linguistic reconstruction that it is seldom if ever explicitly mentioned. Nonetheless, the necessity for some such assumption becomes obvious upon reflecting that if adoption of a theory T were to change the meanings of terms from L_o, then the fact that T logically entails a sentence which is false in L_o could not be held against the truth of T in $L_{(T)}$, and the empiricist would have no way to falsify a theory on the basis of empirical evidence. A stipulation which needs to be made, then, is that the $L_{(T_i)}$ are a family of languages whose observational core is semantically equivalent from one theoretically enriched language to another:

Assumption 6. If S is a sentence in language L_o, then S in $L_{(T_i)}$ is equivalent in meaning to S in $L_{(T_j)}$.

Assumption 7. If a sentence S_1 in $L_{(T_i)}$ is equivalent in meaning to a sentence S_2 in $L_{(T_j)}$, then S_1 in $L_{(T_i)}$ is incompatible with a sentence S_3 in $L_{(T_k)}$ if and only if S_2 in $L_{(T_j)}$ is incompatible with S_3 in $L_{(T_k)}$. In view of Assumptions 6 and 7, incompatibility of sentences in specified languages in family $L_{(T_i)}$ is not essentially relative to those languages when the sentences are also in L_o. Similarly, Assumptions 2, 4, 6 and 7 imply that a sentence common to L_o and $L_{(T)}$ is A-true (inconsistent) in $L_{(T)}$ if and only if it is A-true (inconsistent) in L_o.

We may now define the "observational consequences" [O-consequences] of a theory T to be those analytic consequences of T which are also sentences in L_o. In view of Assumptions 1 and 3, T A-implies S in $L_{(T)}$ (i.e., $T \supset S$ is A-true in

$L_{(T)}$) if and only if $\vdash A_o \cdot T \supset S$ in $L_{(T)}$. Therefore,

Definition 1. Sentence S is an O-consequence of theory $T =_{\text{def}} S$ is a sentence in L_o such that $\vdash A_o \cdot T \supset S$ in $L_{(T)}$.

Definition 2. The Ramsey-sentence $[R_T]$ of a theory $T(\tau_1, \cdots, \tau_n) =_{\text{def}} (\exists \phi_1, \ldots, \phi_n)\, T(\phi_1, \ldots, \phi_n)$. (Strictly speaking, the Ramsey-sentence of a theory is any of a class of formally equivalent sentences, unless we introduce some convention restricting which of various syntactically permissible variables is to replace τ_i in going from T to R_T.) The Ramsey-sentence of a theory is a sentence in L_o which asserts, in effect, that entities satisfying the observational predicate from which the theory is constructed do, in fact, exist. R_T has the important property that not only is it an O-consequence of T, but any O-consequence of T is also an analytic consequence of R_T:

Lemma 1. A sentence S is an O-consequence of theory T if and only if $\vdash A_o \cdot R_T \supset S$ in L_o. Proof: Since $\vdash T \supset R_T$ in $L_{(T)}$, obviously $\vdash A_o \cdot T \supset S$ in $L_{(T)}$ if $\vdash A_o \cdot R_T \supset S$ in L_o. For the converse, let S be a sentence in L_o such that $\vdash A_o \cdot T(\tau_1, \ldots, \tau_n) \supset S$ in $L_{(T)}$. Since S and A_o contain no theoretical terms, it follows by a well-known principle in quantification theory that

$$\vdash \qquad (\exists \phi_1, \ldots, \phi_n)[A_o \cdot T(\phi_1, \ldots, \phi_n)] \supset S$$

in $L_{(T)}$. But A_o contains no variables bound by the quantifiers, so

$$\vdash \qquad A_o \cdot (\exists \phi_1, \ldots, \phi_n)\, T(\phi_1, \ldots, \phi_n) \supset S$$

in $L_{(T)}$ and hence also in L_o, since this formula contains only terms in L_o. QED.

An obvious condition under which two theories T_1 and T_2 are incompatible is that they have incompatible observational consequences. Formalizing this situation demands care, however, for as has already been pointed out, the language which results from acceptance of T_1 is (in general) different from the one which results from acceptance of T_2, and an inter-linguistic comparison is hence necessary.

Definition 3. Theories T_1 and T_2 have incompatible O-consequences $=_{\text{def}}$ There exist O-consequences S_i and S_j of T_1 and T_2, respectively, such that S_i in $L_{(T_1)}$ is incompatible with S_j in $L_{(T_2)}$

Lemma 2. Theories T_1 and T_2 have incompatible O-consequences if and only if there exist O-consequences S_i and S_j of T_1 and T_2, respectively, such that S_i and S_j are incompatible in L_o. Proof: Suppose S_i and S_j are O-consequences of T_1 and T_2, respectively, which are incompatible in L_o. By Assumption 6, S_i in $L_{(T_1)}$ and S_j in $L_{(T_2)}$ are equivalent in meaning, respectively, to S_i and S_j in L_o. Then by Assumption 7, S_i in $L_{(T_1)}$ is incompatible with S_j in L_o, and again by Assumption 7, S_i in $L_{(T_1)}$ is incompatible with S_j in $L_{(T_2)}$. Conversely, suppose that T_1 and T_2 have incompatible O-consequences. Then by definition, there exist O-consequences

S_i and S_j of T_1 and T_2, respectively, such that S_i in $L_{(T_1)}$ is incompatible with S_j in $L_{(T_2)}$. By successive applications of Assumptions 6 and 7, as before, we show that S_i and S_j are incompatible in L_o. QED.

Lemma 3. Two theories have incompatible O-consequences if and only if their Ramsey-sentences are incompatible in L_o. Proof: This follows routinely from Assumptions 1-4 and Lemmas 1 and 2.

Theorem 1. Two theories which have no theoretical terms in common are incompatible only if they have incompatible O-consequences. Proof: Let $T_1(\tau_1, \ldots, \tau_n)$ and $T_2(\mu_1, \ldots, \mu_m)$ be theories such that $\tau_i \neq \mu_j$ for all $i \leq n$ and $j \leq m$. Then in view of Lemma 3, it suffices to prove that the Ramsey-sentences, R_{T_i} and R_{T_2}, of T_1 and T_2 must be incompatible if T_1 and T_2 are. Suppose that T_1 and T_2 are incompatible. Then, by Assumption 5, $T_1 \cdot T_2$ is inconsistent in $L_{(T_1 \cdot T_2)}$, and hence, by Assumptions 1 and 2, $\vdash A_o \cdot T_1 \cdot T_2 \supset \sim (T_1 \cdot T_2)$ in $L_{(T_1 \cdot T_2)}$, from which it follows that $\vdash A_o \supset \sim (T_1 \cdot T_2)$. Rewriting this last step in greater detail, we have

$$A_o \supset \sim [T_1(\tau_1, \ldots, \tau_n) \cdot T_2(\mu_1, \ldots, \mu_m)].$$

Then by simple steps in $L_{(T_1 \cdot T_2)}$, and hence also in L_o, since only terms in L_o are involved,

$$\vdash \quad (\phi_1, \ldots, \phi_{n+m})\{A_o \supset \sim [T_1(\phi_1, \ldots, \phi_n) \cdot T_2(\phi_{n+1}, \ldots, \phi_{n+m})]\}$$
$$\vdash \quad A_o \supset (\phi_1, \ldots, \phi_{n+m}) \supset \sim [T_1(\phi_1, \ldots, \phi_n) \cdot T_2(\phi_{n+1}, \ldots, \phi_{n+m})]$$
$$\vdash \quad A_o \supset \sim (\exists \phi_1, \ldots, \phi_{n+m})[T_1(\phi_1, \ldots, \phi_n) \cdot T_2(\phi_{n+1}, \ldots, \phi_{n+m})]$$
$$\vdash \quad A_o \supset \sim (\exists \phi_1, \ldots, \phi_n)\, T_1(\phi_1, \ldots, \phi_n) \cdot [(\exists \phi_1, \ldots, \phi_m)\, T_2(\phi_1, \ldots, \phi_m)].$$

But this last step is simply $\vdash A_o \supset \sim (R_{T_1} \cdot R_{T_2})$, which shows, by Assumptions 2–4, that R_{T_1} and R_{T_2} are incompatible. QED.

Although the empiricist interpretation of theoretical meanings has been cited to explain why certain moves have *not* been made here, none of the actual Assumptions so far should be offensive to one who questions this view. It is now time to make essential use of the empiricist's position. According to this view, theoretical terms *acquire* whatever meaning they have by comprising part of an accepted theory. As a result, it makes no difference what sign-designs are used for the theoretical terms so long as they are not terms of the observation language and distinctiveness is maintained. Hence two theories which are identical except for their theoretical terms must receive the same meaning when they are (alternately) accepted:

Assumption 8. If $T(\tau_1, \ldots, \tau_n)$ and $T(\tau_1^*, \ldots, \tau_n^*)$ are two theories which are identical except that possibly $\tau_i \neq \tau_i^*$ for $i \leq n$, then $T(\tau_1, \ldots, \tau_n)$ and $T(\tau_1^*, \ldots, \tau_n^*)$ are equivalent in meaning.

Theorem 2. Two theories are incompatible only if they have incompatible O-consequences. Proof: Let $T_1(\tau_1, \ldots, \tau_n)$ $[T_1]$ and $T_2(\tau_1^*, \ldots, \tau_n^*)$ $[T_2^*]$ be theories such that the τ_i are not necessarily distinct from the τ_j^*. Let $T_2(\mu_1, \ldots, \mu_m)$ $[T_2^\mu]$ be formed from T_2^* by replacing each τ_i^* by μ_j, where the μ_j are distinct and such that $\tau_i \neq \mu_j$ for all $i \leq n$ and $j \leq m$. Then by Assumption 8, T_2^μ is equivalent in meaning to T_2^*. But by Lemma 3 and Theorem 1, T_1 and T_2^μ are incompatible only if R_{T_1} and $R_{T_2}\mu$ are incompatible in L_o, so by Assumption 7, T_1 and T_2^* are incompatible only if R_{T_1} and $R_{T_2}\mu$ are incompatible in L_o. But $R_{T_2}\mu = R_{T_2^*}$, and hence, in view of Lemma 3, T_1 and T_2^* are incompatible only if they have incompatible O-consequences. QED.

In particular, two (alternately acceptable) theories $T(\tau_1, \ldots, \tau_n)$ and $\sim T(\tau_1, \ldots, \tau_n)$ are not necessarily incompatible, even though one is prima facie the negation of the other, for the τ_i will (in general) have different meanings in $L_{(T)}$ and $L_{(\sim T)}$.

Since the proofs of Theorems 1 and 2 do not require that $n > 0$, they may also be interpreted as a condition on the incompatibility of a theory and an observation sentence. That a sentence in L_o is incompatible with theory T only if it is incompatible with an O-consequence of T also follows directly from Assumptions 1–4, 6 and 7.

Except for strengthening Theorems 1 and 2 to biconditionals, this is about as far as we can go without penetrating much deeper into the semantics of theoretical terms. However, Theorem 2 has certain implications which, while not rigorous consequences of the Assumptions made so far, seem intuitively inescapable.

Intuitive Corollary 1. A theory is equivalent in meaning to its Ramsey-sentence. Demonstration: Since a theory entails its Ramsey-sentence, the meaning content of T must include that of R_T. On the other hand, any denial of T must also amount to a denial of R_T; for to deny (by assertion or by entailment) a theory T is either to assert an observation sentence which is incompatible with T or to accept another theory which is incompatible with T, and by Lemma 3 and Theorem 2, any theory or observation sentence which is incompatible with T is also incompatible with R_T. But if T were stronger than R_T in its belief commitments, it certainly should be possible to deny T without denying R_T.[5] And if T is as least as strong, but no stronger than R_T in meaning content, then T and R_T must be equivalent in meaning.

Intuitive Corollary 2. Two theories are equivalent in meaning if and only if they have the same observational consequences. Demonstration: Intuitive Corollary 1

[5]Nevertheless, it is possible to construct semantical theories under which T is, in fact, stronger than R_T, even though the difference between T and R_T cannot be extracted for separate denial. Such a situation would arise, for example, if the semantics of theoretical concepts were such that whenever several sets of satisfiers of $T(\phi_1, \ldots, \phi_n)$ exist, the theory $T(\tau_1, \ldots, \tau_n)$ is able to single out just one such set as the referents of the τ_i. For arguments against this latter possibility, however, see Part I of this paper.

implies that two theories T_1 and T_2 are equivalent in meaning if and only if their Ramsey-sentences are analytically equivalent. But it follows from Assumption 3 and Lemma 1 that R_{T_1} and R_{T_2} are analytically equivalent if and only if T_1 and T_2 have the same O-consequences. Hence T_1 and T_2 are equivalent in meaning if and only if they have the same O-consequences.

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