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A Note on Carnap's Meaning Criterion

What the Holy Grail was to the Knights of the Round Table, so has the Meaning Criterion been to Logical Empiricists. And indeed, for one who believes that many of the verbal puzzles which have so vexed philosophers are in fact empty of cognitive significance, there is something exhilarating in the thought of actually finding a test by which can be determined whether or not a statement is factually meaningful. Yet it has repeatedly occurred that no sooner does a philosopher return from his Quest, proudly displaying the Criterion he has discovered, than some churlish critic, by extracting various unseemly consequences, exposes it as counterfeit.[1]

Rudolf Carnap (1956)[2] has recently proposed a test of significance especially designed to do justice to terms introduced by scientific theories. We begin by assuming an "observation language," L_{O}, each term in the descriptive (i.e., extra-logical) primitive vocabulary, V_{O}, of which is cognitively meaningful, and the syntax of which is sufficiently simple that we may take the cognitive significance of (well-formed) sentences in L_{O} for granted. We now introduce a "theoretical language," L_{T}, whose syntax includes but is in principle richer than that of L_{O} and whose primitive descriptive vocabulary, V_{T}, consists of non-observational terms whose meanings derive wholly from terms of V_{O} in a manner to be described, and form the total language, L, whose syntax is that of L_{T} and whose primitive descriptive vocabulary is $V_{\mathrm{O}} + V_{\mathrm{T}}$. The sentences of L may be divided into three categories: (a) those which also belong to L_{O} and whose descriptive terms hence come only from V_{O}; (b) those which also belong to L_{T} and whose descriptive terms hence come only from V_{T}; and (c) "mixed" sentences, namely, those which belong neither to L_{O} nor to L_{T} and which hence either contain terms from both V_{O} and V_{T} or, though containing descriptive terms only from V_{O} are not constructable by the syntax of L_{O}.[3]

Now let T (for "theory") be the conjunction of a set of sentences in L_{T}, and C (for "correspondence rules") be the conjunction of a set of mixed sentences. Then the theory-*cum*-correspondence-rules $T \cdot C$ may convey meaning on some of the terms in V_T, and hence on certain sentences which contain them, by providing them

[1]By no means do I wish to disparage the search for the meaning criterion, having but recently gone aquesting myself (Rozeboom, 1962).

[2]For clarifying the relation of the syntax of the theoretical language to that of the combined observational-theoretical language I have also drawn upon Carnap's as yet unpublished paper, "Carl G. Hempel on Scientific Theories." (Editor: The paper referred to is Carnap (1963)).

[3]These classes are not wholly disjoint, for all L-sentences—i.e., sentences containing only logical terms—in L are sentences of L_{T}, and hence all L-sentences in L_{O} are also in L_{T}.

with suitable connections with terms in V_{O}. The particular conditions proposed by Carnap under which a theoretical term is given meaning by $T \cdot C$ are as follows (p. 51):

"*D1.* A term 'M' is significant relative to the class K of terms, with respect to L_{T}, L_{O}, T, and $C =_{\mathrm{df}}$ the terms of K belong to V_{T}, 'M' belongs to V_{T} but not to K, and there are three sentences, C_{M} and S_{K} in L_{T} and S_{O} in L_{O}, such that the following conditions are fulfilled:

(a) S_{M} contains 'M' as the only descriptive term.

(b) The descriptive terms in S_{K} belong to K.

(c) The conjunction $S_{\mathrm{M}} \cdot S_{\mathrm{K}} \cdot T \cdot C$ is consistent (i.e., not logically false).

(d) S_{O} is logically implied by the conjunction $S_{\mathrm{M}} \cdot S_{\mathrm{K}} \cdot T \cdot C$.

(e) S_{O} is not logically implied by $S_{\mathrm{K}} \cdot T \cdot C \ldots$"

"*D2.* A term 'M_{n}' is significant with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T$, and $C =_{\mathrm{df}}$ there is a sequence of terms 'M_1',...,'M_n' of V_{T}, such that every term 'M_i' $(i = 1, \ldots, n)$ is significant relative to the class of those terms which precede it in the sequence, with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T$, and C."

Now, it would be most gratifying to be able to conclude that the long-sought answer to the problem of significance has at last been found. Still, questing is not always such an unpleasant chore, and it would spoil the fun for the rest of us if Carnap's proposed criterion really were thoroughly satisfactory. So it is with mixed feelings that I call attention to two consequences of *D1* and *D2* which seem to me to weigh seriously against their acceptability. In brief, (I) if *D1* and *D2* are to work, we must define "logical truth" ("L-truth") in such a way that every sentence containing only logical terms is either L-true or L-false, and (II) a theoretical term which is significant relative to a given theory can be made to lose its significance by enrichment of the theory.

I

Let a sentence which contains no descriptive terms be called an "L-sentence"; let a language L_i be called "L-closed" or "L-open," respectively, according to whether or not every L-sentence in L_i is either logically true or logically false; and let L_i be called "L-closed relative to S" or "L-open relative to S," respectively, according to whether or not S is a sentence of L_i such that for every L-sentence S_{L} in L_i, either S logically implies S_{L} or S logically implies $\sim S_{\mathrm{L}}$. It is clear that if L_i is not L-closed, a sentence S of L_i unless inconsistent, will have to have very special properties in order that L_i be L-closed relative to S; hence if L_i is L-open, we may also expect it to remain L-open relative to a given S so long as S is consistent and has not been specifically selected through considerations of L-closure. In fact, if our definition of L-truth permits a language to be L-open, we may well expect to be able to prove, similar to Gödel's incompleteness theorem, that a sufficiently rich L-open language is also L-open relative to any consistent sentence of the language. (This result follows immediately from Gödel's theorem and Henkin's results (Henkin, 1950) if we define "logical truth" as "true in all Henkinian models.")

According to *D1* and *D2*, if the theory-*cum*-correspondence-rules $T \cdot C$ gives meaning to any theoretical terms at all, there is a sentence $S_{\mathrm{M}}1$ in L_{T} whose only descriptive term is a theoretical term 'M_1,' and a sentence S_{O} in L_{O} such that $S_{\mathrm{M}}1 \cdot T \cdot C$ is consistent and logically implies S_{O}, and $T \cdot C$ does not logically imply S_{O}. Suppose, now, that language L is L-open relative to $S_{\mathrm{M}}1 \cdot T \cdot C$. Then there is a sentence S_{L} of L such that S_{L} contains no descriptive terms and $S_{\mathrm{M}}1 \cdot T \cdot C$ logically implies neither S_{L} nor $\sim S_{\mathrm{L}}$. Noting that $S_{\mathrm{L}} \cdot T \cdot C$ and $\sim S_{\mathrm{L}} \cdot T \cdot C$ do not both logically imply S_{O} (since otherwise, contrary to hypothesis, $T \cdot C$ would also logically imply S_{O}), let S_{L}^* be S_{L} unless $\sim S_{\mathrm{L}} \cdot T \cdot C$ logically implies S_{O}, in which case S_{L}^* is $\sim S_{\mathrm{L}}$. (This ensures that $\sim S_{\mathrm{L}}^* \cdot T \cdot C$ does not entail S_{O} and hence that (e) obtains below.) Now let $S_{\mathrm{M}}\mathrm{i}$ be the sentence '$M_i = M_i$,' where 'M_i' is any theoretical term in V_{T} other than 'M_1,' and consider the sentences $S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{L}}^*$ and $S_{\mathrm{L}}^* \supset S_{\mathrm{M}}1$, both of which are in L_{T}. We can easily show that (a) $S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{L}}^*$ contains 'M_i' as its only descriptive term; (b) the only descriptive term in $S_{\mathrm{L}}^* \supset S_{\mathrm{M}}1$ is 'M_1,' which by hypothesis is significant with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T,$ and C; (c) the conjunction $(S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{L}}^*) \cdot (S_{\mathrm{L}}^* \supset S_{\mathrm{M}}1) \cdot T \cdot C$ is consistent; (d) S_{O} is logically implied by $(S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{L}}^*) \cdot (S_{\mathrm{L}}^* \supset S_{\mathrm{M}}1) \cdot T \cdot C$; and (e) S_{O} is not logically implied by $(S_{\mathrm{L}}^* \supset S_{\mathrm{M}}1) \cdot T \cdot C$. Hence by *D1* and *D2*, 'M_{i}' is significant with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T,$ and C.

We thus see that according to Carnap's proposed criterion, *any* term in V_{T}—the term does not even have to be in $T \cdot C$—is significant with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T,$ and C so long as $T \cdot C$ contains a theoretical term whose significance is attested by S_{M}, and L is L-open relative to $S_{\mathrm{M}} \cdot T \cdot C$; and if L is L-open, it will also be L-open relative to $S_{\mathrm{M}} \cdot T \cdot C$ for most if not all S_{M} and $T \cdot C$. The difficulty is even more apparent (though *D1* can be doctored up to avoid this latter case) if there is an L-sentence, S_{OL}, in L_{O} such that neither S_{OL} nor $\sim S_{\mathrm{OL}}$ is L-implied by $T \cdot C$. For then, letting $S_{\mathrm{M}}\mathrm{i}$ be '$M_i = M_i$' for any theoretical term 'M_i' and any consistent $T \cdot C$, S_{OL} is a sentence in L_{O} logically implied by the consistent conjunction $(S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{OL}}) \cdot T \cdot C$ but not by $T \cdot C$, while $S_{\mathrm{M}}\mathrm{i} \cdot S_{\mathrm{OL}}$ is a sentence in L_{T} containing 'M_i' as its only descriptive constant. Hence by *D1* and *D2*, when such an S_{OL} exists, every term in V_{T} is significant with respect to $L_{\mathrm{O}}, L_{\mathrm{T}}$, T, and C. We must conclude that Carnap's criterion is wholly unacceptable for an L-open language.

The force of this objection depends, of course, upon the extent to which we expect to encounter L-open languages—i.e., whether or not we admit the possibility that a sentence containing only logical terms might be neither logically true nor logically false. Carnap's own recent proposals for L-truth (e.g., in *Meaning and Necessity*), namely, that a sentence of L is L-true if and only if it is true in all state-descriptions in L, does, in fact, entail that languages to which the definition applies are L-closed. However, serious objections can be raised against this particular interpretation of L-truth,[4] while it is by no means the case that every reasonable definition of L-truth that might be offered yields L-closure. A discussion of logical truth is far beyond the scope of this note. Nonetheless, it may seriously be doubted whether any definition which entails that every L-sentence is either L-true or L-false could be tolerated by our intuitive concept of logical truth. For example, whether the number of particulars in existence is finite (and if so, how many), denumerably infinite, or of a higher trans-finitude, is surely not a problem which can be settled on logical grounds alone. Yet if '=' may be construed as a logical term, the various possible answers to this question can be expressed wholly in logical terms—e.g., if 'x', 'y', and 'z' range over particulars, '$(\exists x)(\exists y)[x \neq y \cdot (z)(z = x \vee z = y)]$' is true if and only if there are exactly two particulars. In short, whether or not we shall eventually wish to define L-truth in such a way that every language is L-closed, it seems to me that the matter is a highly controversial one, and I submit that any meaning criterion which is at best acceptable only for an L-closed language must be viewed with grave suspicion.

II

Any intuitively acceptable meaning criterion must surely have the property that if a term 'M' is significant with respect to $L_{\mathrm{T}}, L_0, T,$ and C, and $T' \cdot C'$ differs from $T \cdot C$ only in containing additional T-postulates or C-postulates, then 'M' is also significant with respect to $L_{\mathrm{T}}, L_0, T',$ and C', so long as $T' \cdot C'$ is consistent. For if a theory-*cum*-correspondence-rules confers meaning on a theoretical term by supplying it with sufficiently strong "connections" with L_{O}, addition of new postulates does nothing to weaken these connections unless the additions make the theory inconsistent, in which case the old connections are weakened in the sense that they no longer matter. But Carnap's criterion does *not* have this property—a theoretical term may be made to *lose* its significance by adding new postulates, even though the consistency requirement is not violated. To see this

[4]For example, it is assumed that the truth of every non-atomic sentence in L is determined wholly by the truths of atomic sentences in L. It follows that no bound variable in L can have in its range an entity which is not designated by an expression in the language, and hence that the range of every bound variable in L is denumerable.

in a general way, observe that according to *D1* and *D2*, if 'M' is significant, there must be sentences $S_{\mathrm{M}}, S_{\mathrm{K}},$ and S_{O} such that $S_{\mathrm{M}} \cdot S_{\mathrm{K}} \cdot T \cdot C$ logically implies S_{O}, while $S_{\mathrm{K}} \cdot T \cdot C$ does *not* logically imply S_{O}. But if $S_{\mathrm{M}}, S_{\mathrm{K}},$ and S_{O} attest the significance of 'M' with respect to $L_{\mathrm{T}}, L_0, T,$ and$C$ in this way, they may no longer do so with respect to L_{T}, L_0, T', and C' when T' and C' are formed from T and C by (consistent) addition of new postulates (since $S_{\mathrm{K}} \cdot T' \cdot C'$ may logically imply S_{O} when $S_{\mathrm{K}} \cdot T \cdot C$ does not), and there may not be any other sentences $S'_{\mathrm{M}}, S'_{\mathrm{K}},$ and S'_{O} which show 'M' to be significant with respect to $L_{\mathrm{T}}, L_{\mathrm{O}}, T'$, and C'.

This point may be made more rigorously as follows: Let a theory-*cum*-correspondence-rules $T \cdot C$ be called "maximally L_{O}-consistent" when $T \cdot C$ is consistent and such that if S is any sentence in L_{O} not logically implied by $T \cdot C$, $T \cdot C \cdot S$ is inconsistent. (When L_{O} is simple, or when L_{T} is sufficiently richer syntactically than L_{O}, there is no reason in principle why there may not be maximally L_{O}-consistent theories.) But by *D1* and *D2*, there is no theoretical term which is significant with respect to a maximally L_{O}-consistent theory. For if $T \cdot C$ is such a theory, then for every sentence S_{O} in L_{O}, $T \cdot C$ logically implies either S_{O} or $\sim S_{\mathrm{O}}$, and hence for any $S_{\mathrm{M}}, S_{\mathrm{K}}$, and S_{O}, either condition (c), (d), or (e) of *D1* must be violated. Hence according to Carnap's criterion, the significant theoretical terms in a theory may be deprived of significance by enriching the theory to maximal L_{O}-consistency. And while there is perhaps no *logical* reason why a theoretical term, if significant with respect to a given theory, should remain significant under (consistent) addition of further postulates, it nonetheless seems to me that our intuitive feelings here are sufficiently strong to make any meaning criterion which does not preserve significance under theory enrichment highly suspect.

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