Published in *The Philosophical Review*, 1958, *57*(3), 353–366.

The Logic of Color Words

An altercation between Hilary Putnam and Arthur Pap has recently appeared in the *Philosophical Review* concerning the "good, old-fashioned" analyticity of color incompatibility statements such as "Nothing can be both red and green all over at once." Starting with the primitive relation, "x is indistinguishable in color from y," ["I(x,y)"], Putnam (1956) constructs the reflexive, symmetric, and transitive relation, "x is exactly the same color as y" ["E(x,y)"] by the stipulative definition,

$$\mathrm{D}_1 \qquad \mathrm{E}(x,y) =_{\mathrm{def}} (z)[\,\mathrm{I}(z,x) \equiv \mathrm{I}(z,y)].$$

From D_1 and the further definitions of "F is a color" ["Col (F)"]

$$\mathrm{D}_2 \qquad \mathrm{Col}(F) =_{\mathrm{def}} (\exists y)(x)[F(x) \equiv \mathrm{E}(x,y)],$$

he is able to deduce, by straightforward logical procedures, that nothing is two different shades of color at once—that is, that

$$\mathrm{T} \qquad (F)(G)(x)(\mathrm{Col}[F] \cdot \mathrm{Col}[G] \cdot [F \neq G] \supset \sim [F(x) \cdot G(x)]).$$

Pap (1957), in turn, while accepting the bulk of Putnam's analysis, points to the need for certain qualifications (principally, in regard to whether "Red is a color" may be treated as analytic in the strict sense) to which Putnam (1957) acquiesces.

Now, it is no doubt presumptuous of me to intrude into a private argument which, moreover, appears resolved to the mutual satisfaction of both parties. Yet I feel that Putnam's analysis is both too ingenious and too seductively misleading to be put aside without further exploration. I shall attempt to make the following points: I. Putnam's definition of "E(x,y)" [D_1] does not adequately analyze our intuitive notion of "x is exactly the same color as y" and indeed falls short of his own criteria for such an explication. II. Putnam's analysis of color predicates cannot satisfactorily be extended to the general case of multiple coloration. III. The claim that nothing can be two different colors all over at the same time is true only when sufficiently qualified in a way that *does* make it good, old-fashioned analytic, and in demonstrating this we seem forced to recognize that there are two basically different ways in which color predicates can be applied to objects.

I

As Putnam has properly assumed, the relational predicate, "x is exactly the same color as y" is an important color concept which, given a satisfactory analysis, might be expected to perform heroically in the explication of further color concepts. We may also agree that, while indeed difficult to pin down, "x is exactly the same color as y" does seem to be a stronger, transitive version of "x is indistinguishable in color from y" which is intuitively reflexive and symmetric, but demonstratively not transitive. Therefore we may accept Putnam's stipulation that any construction acceptable as an analysis of "E(x, y)" must (a) be reflexive,[1] symmetric, and transitive; and (b) must analytically (in either the strict or the broad sense) entail "I(x, y)." For otherwise we should be able to imagine a world in which "$(x)(y)[\mathrm{E}(x,y) \supset \mathrm{I}(x,y)]$" did not obtain, so that "$\mathrm{E}(x,y)$" could not be said to be a stronger version of "$\mathrm{I}(x,y)$" in any meaningful sense.

But one begins to wonder whether the "meaning content" of "E(x, y)," as explicated by D_1, really does include that of "I(x, y)" when one observes that the reflexivity, symmetry, and transitivity of "E(x, y)" follow analytically from D_1, irrespective of the reflexivity and symmetry of "I(x, y)." It does not seem unfair to be suspicious of so much received at so little expense, and this suspicion becomes strengthened upon realization that contrary to stipulation (b), "E(x, y)" does *not* entail "I(x, y)," but only "$\mathrm{I}(x,\ x) \equiv \mathrm{I}(x,\ y)$." To be sure, we can derive "I(x, y)" from "E(x, y)" and "I(x, x)," while "$(x)[\mathrm{I}(x,\ x)]$" may be taken as a meaning postulate, but the fact that "I(x, x)" has to be adduced in *addition* to the analysis of "E(x, y)" for the derivation of "I(x, y)" shows that the material implication of "I(x, y)" by "E(x, y)," defined by D_1 is formally contingent, not analytic. Nor does it seem correct to say that "I(x, y)" can be obtained as a component in a meaning analysis of "E(x, y)," for this is to imply that a statement of form "$(x)[\Phi(x) \equiv \Psi(x)]$" has "$\Phi(a)$" as part of its meaning content for some individual, a.

The fact that "E(x, y)" defined as in D_1, is actually *weaker* than "I(x, y)" in meaning content accounts, I think, for the "paradox" which Pap constructs from D_1 and D_2. Pap (1957, p. 96) shows that if "red"—referring to a specific shade of red, say red_6—be defined as the color of some red object, a, by the definition, "$\mathrm{Red}_6(x) =_{\mathrm{def}} \mathrm{E}(x,a)$" then "$\mathrm{Red}_6(a)$" becomes an analytic truth in the strict sense, and the "formal definitions therefore warrant an "ontological leap" to the existence ... of red ... things." Apparently accepting the adequacy of D_1, Pap goes on to argue that such a "startling" consequence challenges the presumption that color predicates can be defined in this way. Putnam (1957, p. 102), in turn,

[1] Actually, the reflexivity of "I(x, y)" is not as intuitive as it might appear, for the claim, e.g., that a tone is indistinguishable in color from itself should make one pause for thought. However, it is even stranger to assert that a tone is not indistinguishable in color from itself, which is our only alternative unless we regard the expression as meaningless. While sentences which, though syntactically well formed, are intuitively distressing are not infrequently written off as meaningless, such arbitrary tactics find me wholly unresponsive. Hence I, at least, am willing to admit that color indistinguishability is a reflexive relation, whether applied to colored objects or not.

concedes that this is indeed a deplorable outcome and retreats to interpreting specific color names as primitive terms. Now I shall argue later (in II) that D_2, which neither Pap nor Putnam wishes to abandon, does in fact commit us to defining specific color predicates in terms of color-identities with prototypical particulars. But such a definition in itself, even though it does involve us in an "ontological leap" of a sort, does so in a very inoffensive way, not at all peculiar to the present situation. As a preliminary, note that we are now referring to a specific shade of red, rather than to a generic grouping of the various reds. Therefore, Pap's argument that we abstract color qualities from *several* particulars may be irrelevant here, for the color terms of ordinary language seem to refer to classes of specific shades, whereas it is not at all implausible that we learn a specific shade by ostensive definition based on a single particular. And if by "$\mathrm{Red}_6(x)$" we really *do* mean "x is exactly the same color as a," then "$\mathrm{Red}_6(a)$" has exactly the same logical status as does "The standard meter bar in Paris is exactly one meter long." To be sure, neither of these, when "being the same color as" or "having the same length as" is taken as primitive, or is constructed from definite descriptions in which "color" and "length" are primitive, is strictly speaking analytic in the narrow sense (though, to borrow Putnam's phrase, they certainly *feel* analytic). Thus when "$\mathrm{E}(x,\ y)$" is taken as primitive, deduction of "$\mathrm{Red}_6(a)$" requires the meaning postulate that "$\mathrm{E}(x,\ y)$" is reflexive. But here is not where the "ontological leap" contained in "$\mathrm{Red}_6(a)$" lies, for "$(x)[\mathrm{E}(x,x)]$" involves no ontological commitments. Rather, the damage is done by admission of "a" as a proper name or description [compare the deduction of "$(\exists x)(x = a)$" from "$(x)(x = x)$"].[2] So it is not the ontological commitments of "$\mathrm{Red}_6(a)$," when "$\mathrm{Red}_6(x)$" is defined as "$\mathrm{E}(x,\ a)$," that should bother us, but rather its stark, uncompromising tautologicality under D_1. To say that a is red_6 in color, or that the standard meter bar is one meter long, is surely to say *something*, if only no more than what is asserted by instantiation of a meaning rule. But if "$\mathrm{Red}_6(a)$" is reduced by D_1 to its ultimate definiens, we arrive at "$(x)[\mathrm{I}(x,a) \equiv \mathrm{I}(x,a)]$" which says nothing factual whatsoever. Thus D_1 must be inadequate as an analysis of "x is exactly the same color as y."

Now the objection raised so far to D_1 can be met very simply. We need but conjoin "$\mathrm{I}(x,\ y) \cdot \mathrm{I}(y,\ x)$" to the definiens in D_1:

D_1' $\qquad \mathrm{E}(x,y) =_{def} \mathrm{I}(x,y) \cdot \mathrm{I}(y,x) \cdot (z)[\mathrm{I}(z,x) \equiv \mathrm{I}(z,y)],$

which is analytically symmetric and transitive, and hence suffices to prove theorem T. Note that "$\mathrm{Red}_6(x) =_{\mathrm{def}} \mathrm{E}(x,a)$" no longer gives "Red (a)" as narrowly analytic, for under D_1' "Red (a)" entails "$\mathrm{I}(a,\ a)$," and the latter can be obtained

[2] Even admission of $\mathrm{Red}_6(x)$" as a primitive term involves us in an ontological problem. For then, from "'$(\Phi)(\Phi = \Phi)$," we can prove "$(\exists\Phi)(\Phi = \mathrm{Red}_6)$"—i.e., that there exists something which is identical with red_6-ness. But if $\mathrm{red}_6 -$ ness has no exemplification, its "existence" is odd indeed.

only from a formally contingent meaning rule about the reflexivity of the primitive, "$\mathrm{I}(x,\ y)$." It is very dubious, however, whether *any* expression in which "$\mathrm{I}(x,\ y)$" occurs is acceptable as an explication of "$\mathrm{E}(x,\ y)$." For "$\mathrm{I}(x,\ y)$" is based upon a triadic (or higher) relation involving two colored objects *and a discriminator*, and the logical complexities into which this gets us surely need not be spelled out here. Nor can we argue that use. of "$\mathrm{I}(x,\ y)$" gives a phenomenalistic reduction to "$\mathrm{E}(x,\ y)$," for "x is indistinguishable from y" is a dispositional predicate.

II

But let us press onward, for more exciting game is afoot. Suppose that D_1 *is* acceptable as a definition of "$\mathrm{E}(x,\ y)$," or that "$\mathrm{E}(x,\ y)$" is simply taken as a primitively stronger version of "$\mathrm{I}(x,\ y)$." We may now inquire whether D_2 is acceptable as an analysis of "F is a color." It will be noticed that D_2 is actually two stipulations neatly condensed into one: (a) it determines when a quality, F, may be called a "color" quality, and (b) it describes the conditions under which a given specific shade of color may be ascribed to a given object. I shall now argue that D_2 is inadequate on both counts. Putnam has shrewdly restricted his analysis to cases of homogeneous coloration "for the sake of simplicity." But examination of D_2 with its contextual restrictions made explicit demonstrates its inapplicability to the general case.

What can we say about the color of a checkerboard? Well, we would be unhappy if we were forced to deny that it is red *and* black—not just red-and-black in the way that we might say a grey object is at the intersection of the blacks and the whites, but two different *specific* shades. Yet as it stands, D_2 and the transitivity of "$\mathrm{E}(x,\ y)$" ineluctably bar such a dual color ascription. Since Putnam reassures us that his analysis is restricted to uniformly colored objects, D_2 stands revealed as an ellipsis in which lies hidden the property of being uniformly colored. Let "x is uniformly colored" be abbreviated "$\mathrm{U}(x)$." Then unpacking D_2 appears to give

$$\mathrm{D}_2' \qquad \mathrm{Col}(F) =_{\mathrm{def}} (\exists y)(x)[\mathrm{U}(y) \cdot (\mathrm{U}[x] \supset [\mathrm{F}(x) \equiv \mathrm{E}(x, y)])] \,,$$

which leaves open the application of color predicates to multicolored objects.

But D_2' will not do at all as a definition of "F is a color." To see why, let us first examine D_2 more closely. Pap (1957, p. 94) has argued convincingly that the biconditional in D_2 must be interpreted as *intensional* rather than material equivalence. That is, D_2 admits a predicate, "F," as a color name if and only if there is some object, y, for which "*F(x)*" is intensionally equivalent to "$\mathrm{E}(x,\ y)$." But since the meaning of "$\mathrm{E}(x,\ y)$" has already been stipulated in terms of "$\mathrm{I}(x,\ y)$," which is primitive, then either "*F(x)*" must be *defined* in terms of "E*(x, y)*," or of "$\mathrm{I}(x,\ y)$," or we have smuggled meaning rules (or worse) into D_2 as a component of the definiens, thereby obtaining the strict analyticity of theorem T only by building the synthetic a priori into the very concept of "color." (For this reason I contended above that Putnam appears committed to *defining* colors as color-identities with particulars.) At any rate, whether we insist that colors be defined in this way or allow a broader kind of intensional equivalence, D_1 would seem to state that to be a color predicate is to *mean* being exactly the same color as some specified object.[3]

But the picture changes when the contextual restrictions of D_2 are made explicit in D'_2. For while "$\mathrm{U}(y)$" merely limits colors to definition in terms of uniformly colored objects, "$\mathrm{U}(x)$" says that for any object x, the (broad or strict) analytic equivalence of "$F(x)$" and "$\mathrm{E}(x, y)$" is *contingent* upon x's being uniformly colored. That is, D'_2 (which is D_2 spelled out) admits as a color predicate *any* F, no matter what its total meaning content, which is constructed in such a way that for some uniformly colored object y, "$F(x) \equiv \mathrm{E}(x, y)$" may be logically deduced from "$\mathrm{U}(x)$." But this admits predicates such as "Either x is exactly the same color as a, or x is a multi-colored baboon" which qualifies as a color predicate under D'_2 so long as a is uniformly colored. Hence D'_2 cannot itself be regarded as a *definition* of "F is a color," at least if "$\mathrm{Col}(F)$" is to retain any pretense at analyzing our intuitive notion, and must at best be replaced by

$$\mathrm{P}_1 \qquad \mathrm{Col}(F) \supset (\exists y)(x)[\mathrm{U}(y) \cdot (\mathrm{U}[x] \supset [F(x) \equiv \mathrm{E}(x, y)])] \,.$$

What, now, is the status of P_1? It is still adequate to prove theorem T for uniformly colored objects, but unless we are actually able to construct a satisfactory definition for "$\mathrm{Col}(F)$" which entails P_1 the latter simply stands as a postulate or meaning rule, and T can no longer be claimed to be good, old-fashioned analytic.

We can go a long way toward defining color predications for non-uniformly colored objects by constructing a definition for "$\mathrm{U}(x)$," a term which appears particularly susceptible to analysis. To say that x is uniformly colored is to say that x is *the same color all over.* Thus given the relation, "x is a part of y" ["$\mathrm{P}(x, y)$"], which may be taken as primitive, or derived from a set-theoretical construction of surfaces, to say that x is uniformly colored is to say that all its parts are exactly the same color—that is,

$$\mathrm{D}_3 \qquad \mathrm{U}(x) =_{\mathrm{def}} (y)(z)[\mathrm{P}(y, z) \cdot \mathrm{P}(z, x) \supset \mathrm{E}(y, z)].$$

We may then say, in general, that if a color may be ascribed to a *part* of an object, it may also legitimately be ascribed to that object as a whole; namely,

$$\mathrm{P}_2 \qquad (F)(x)(y)[\mathrm{Col}(F) \cdot F(x) \cdot \mathrm{P}(x, y) \supset F(y)].$$

[3] We cannot accept Putnam's suggestion that "$\mathrm{Col}(F)$" is to be translated, not "F is a color," but "F is co-extensive with a color," for then theorem T would leave open the possibility there might exist two different colors which are co-extensive and for which the principle of color incompatibility does not hold.

If, now, P_1 and P_2 were the only conditions under which color ascriptions could arise, we could define "F is a color" by

$$D_4 \qquad \mathrm{Col}(F) =_{\mathrm{def}} (\exists y)(x)[\mathrm{U}(y) \cdot (F[x] \equiv (\exists z)[\mathrm{P}(z,x) \cdot \mathrm{E}(z,y)])] \,,$$

from which, given the transitivity of "$\mathrm{P}(x, y)$," both P_1 and P_2 may be deduced. D_4 states that any property is a color if and only if it is coextensive with a class, C, defined by the schema: an object is a member of C if and only if it contains a part which is exactly the same color as the uniformly colored object a. (Of course, Pap's criticism of the extensionality of D_2 applies here also, and the biconditional in D_4 is more properly understood as an intensional equivalence.) Thus D_4 exactly preserves the spirit of D_2, but extends color ascriptions to multicolored objects.

But we are not out of the woods yet, for aspects of our intuitive attribution of colors to objects still remain to be explicated. It will be noted that we have not restricted the range of "x is exactly the same color as y" to uniformly colored objects—otherwise D_3 would be circular. And of course it makes perfectly good intuitive sense to say, for example, that two checkerboards are exactly the same color or, at least, that they are colored exactly alike. We must therefore be prepared to cope with propositions about the color-samenesses of multicolored objects. In particular, "If x is colored F and x is exactly the same color as y, then y also is colored F," and "If x is uniformly colored and x is exactly the same color as y, then y is uniformly colored" would seem to be two truths just as apodictic as the classic case of color incompatibility. But counter-examples may be constructed which show that these propositions *cannot* be derived solely from D_1, D_3, and D_4, even in conjunction with axiomatization of "$\mathrm{P}(x, y)$." A more extensive construction is called for.

Even in its present generality, moreover, D_4 does not extend color predication as far as is our intuitive wont. For under D_4, if an object is to possess a color, it must have at least one part which is exactly the same color as some uniformly colored object. But this is a strong stipulation. What about an object whose color varies continuously over its entire surface? It is difficult to say that such an object has any part whose color is exactly equal to that of some uniformly colored object, yet we would still not hesitate to say such an object is *colored*, albeit we might have trouble saying *what* its color is.

Finally, not only is D_4 too narrow for some purposes, it is also too broad in other respects. For just as its predecessor D_2' did before it, D_4 admits predicates which would never be considered color terms. So long as "a" is the name or description of *any* particular not multicolored, "$\mathrm{E}(x, a)$" satisfies the definiens in D_4 whether a has a color or not, for if a has no color at all, it will satisfy "$\mathrm{U}(x)$" trivially. Thus "x is exactly the same color as this B^b" qualifies under D_4 as a color predicate. Under D_4, *every* particular which does not vary continuously in color is colored; what we normally conceive as the lack of color becomes merely one of the color qualities. (This objection also applies to D_2, with or without contextual restrictions.)

III

Despite the vicissitudes which the line of analysis invoked by Putnam seems to encounter, it is far too soon to abandon hope for an analytic derivation of color incompatibility as expressed in theorem T. It seems to me—and this is my primary motive in exploring Putnam's construction so extensively—that we are now in position to achieve an entirely new (at least to me) insight into the use of color words. It appears that in natural language color qualities are ascribed to objects in two importantly different ways, one of which leaves theorem T empirically false, and the other which *does* make T analytic in the strict sense.

If one transcribes T into ordinary English, bearing in mind its contextual restrictions, one obtains, "If F and G are two different *specific shades* of color, nothing can be (at once) both F and G *all over*." The necessity for the qualification "all over" is obvious—it rules out the case where we predicate different colors of a single object whose parts are not all colored exactly alike. The need for "specific shades" is more subtle. The prima facie explanation is that it merely rules out F and G as the disjunctions of different classes of colors, in the sense in which we use "green" to mean any of a class of shades of green, for in this case the color of an object could be both F and G if these are the disjunctions of overlapping classes. Actually, to say that F is a specific shade of color is meant to have more force than this. It is intended to restrict T to the first of two ways in which F can be predicated of an object, one of which is *explicitly constructed* to let only one specific color be predicated of a uniformly colored object, whereas the other *does* permit a uniformly colored object to be of two different specific shades at once.

We have already seen that it seems perfectly acceptable, even though F and G are different specific shades of color, to say that an object is *both* F and G, so long as the object has differently colored parts. Very well, then, let us construct a series of multicolored objects, each member of which is constructed from its predecessor by modifying the coloration of the latter in a specified way. For the first member of the series, we take, say, a square whose coloration divides it into four vertical stripes of equal width, alternately black and white. Let the rule of the series be that a member is generated by dividing each black stripe of its predecessor into two equally wide vertical stripes, leaving the right hand one of these black and recoloring the other white. Thus each member of the series is white except for two black stripes which diminish in width to vanishing as the series progresses. What, now, are we to say about the coloration of the series' *limit*? Each member of the series is both white and black, and yet the limit is uniformly white. The answer in this case, of course, is simple. For we do not merely say of each series member that it is black and white, we say that it is $p\%$ black and $100 - p\%$ white. The limiting case may be said to be black and white only in the sense that it is 0% black and 100% white. And there is no difficulty in reconciling "x is 0% black" with "x is not black."

But what if we construct our series differently? Let the first member be the same as before, but this time let a member be formed by dividing *each* (black or white) stripe of its predecessor into two equally wide vertical stripes, coloring the right hand one black and the other white. Then each member of the series is characterized by alternating black and white stripes of equal width, with the width of the stripes decreasing to zero as the series progresses. And what are we to say of the coloration of the limiting case now? For the limiting case is obviously a uniform specific shade of *grey*, yet the predicate, "x is 50% white and 50% black" applies to every member of the series. It will not help much to reason that the *limit* of a series is not itself a *member* of the series and hence that while every member of the series is half white and half black, this does not mean that the limiting uniform grey must be called half black and half white. To argue thus is to introduce conceptual discontinuity where analysis finds none in fact. Further, there will be a finite cutting point in the series beyond which its members will be indistinguishable from the limiting grey—in fact, depending upon the further analysis of "E(x, y)," we could wonder whether the series might not even reach a member which is *exactly* the same color as grey. To reach closure on this point, we shall need some postulates about the divisibility of surfaces and the size limits to which color predicates are applicable, but it seems to me that we are inexorably driven to admit that if we can predicate "grey" of a uniformly colored object, we are also justified in saying that it is also half black and half white (or perhaps some other mixture of black and white).

The extension of this line of analysis to the general case is obvious. It is probable that many, perhaps most, uniform specific shades of color can be produced as a limiting mixture of two or more other specific shades of color, where the proportion of a contributing color does not vanish in the limit. If A and B are different uniform specific shades of color which non-vanishingly intermix by some sequence the limit of which is the uniform specific shade of color C, it would seem that if an object x is colored C, there is *some* sense in which it is also proper to say that x is *also* colored A and B.[4] I wonder whether conceivably the reason we class a

[4] This need not be as drastic a conclusion as it might seem. In view of saturation effects, etc., there are probably definite limits to the way in which one shade of color may be produced out of others. Note also that the mixing operations we have constructed here are *conceptual*, not *physical*. The fact that two pigments can be stirred together to give a third color is not germane for present purposes, since this is an empirical law about temporally successive events. But series such as the ones we have constructed reveal timeless, intrinsic relations and bring out the force of our color concepts.

number of specific colors as, for example, "shades of *red*" might not be *because* the specific colors so classified can be compounded out of a prototypical shade of red; that we say a chartreuse object is also green and yellow because we can actually see chartreuse *as* a blend of yellow and green—that is, as *analyzable* into a yellow and a green without necessarily committing ourselves to which green and yellow.

But although I think our use of color terms for multicolored objects does allow ascription of more than one specific shade of color to a uniformly colored object, I would nonetheless agree that this is not the usual sense of "x is of specific shade of color F" when x is uniformly colored. When we say of x that it is red_6, we do not usually mean that red_6 shows forth in x among other shades, but that red_6 is the final composite of whatever shades may be abstracted from x. Thus while a uniformly chartreuse object is also in one sense prototypical green, it is in a different and more customary sense emphatically *not* prototypical green. It is in this second sense of being of a specific shade of color all over that theorem T is true, and moreover, good, old-fashioned analytically true.

To see how this comes about, observe first that full description of the color properties of a multicolored object requires not merely the listing of its colors, but their amount and distribution as well. The second black-and-white series we considered above reveals that patterning is a crucial ingredient in the composition of coloration, for it was by successive redistributions of the same amount of black and white that we generated a uniform shade, grey. In fact, examples such as this show that *just as the uniform shades of color fall on a multi-dimensional continuum, so do color configurations form an extension of that continuum*, an extension which is no less a legitimate part of the *color* continuum than the subspace containing the uniform shades. It seems to me, then, that "specific shade of color" has *patterning* built into it in addition to hue, saturation, and brightness, and that to say "x is of specific shade of color F" in the first sense is to say that F can be abstracted from the color properties of x, whereas to say that x is F in the second, more customary sense, is to say that F is the *totality* of x's color properties. Thus, in the first sense, a uniformly chartreuse object is also both green and yellow just as a checkerboard is both red and black. But also, just as a uniformly chartreuse object is neither (prototypical) green nor yellow in the second sense, neither is a checkerboard either red or black, for these terms apply (in the second sense) to uniform coloration.[5]

And how are we to analyze the property of being a specific shade of color in the second sense? The considerations just advanced expose a prerequisite for any acceptable explication. For if to say that x is F in the second sense is to describe the

[5]Note that there is nothing paradoxical about a uniform specific shade constituting a coloration totality for one object while forming only part of a more complex configuration or a component in another uniform specific shade for another object. Whether there are any specific shades of color which are "basic" in the sense of being unanalyzable into other shades is problematic.

totality of x's color properties, then to say that x is also G, where "G" designates a totality of color properties different from F, is to utter a contradiction. It is a logical part of the concept of "totality" that describing F as a totality of color properties excludes the simultaneous possession of any other total set of color properties. (Compare "No class can have both a total of five members and a total of four members.") Therefore, if we translate "Col(F)" in theorem T as "F is a totality of color attributes," *it would seem to be a necessary condition for any acceptable explication of* "Col(F)" *that theorem T be revealed as analytic.*

As a matter of fact, if we are willing to take "F is a color attribute" in the first sense as a primitive second level property, it is not difficult to construct a definition for "F is a coloration-totality" which is both intuitively plausible and which logically entails T. Let "F is a color attribute" be abbreviated "Ca(F)" and "F is a coloration totality" as "Col(F)." One definition of individual coloration totalities which immediately comes to mind is by reference to particulars through the relation, "x is exactly the same color as y"; for it is intuitively obvious that two objects are exactly alike in coloration if and only if they have the same coloration totality. Since to say that two objects have the same coloration totality is to say that any color attribute of one is also an attribute of the other, we may define "E(x, y)" by

$$\mathrm{D}_1'' \qquad \mathrm{E}(x,y) =_{\mathrm{def}} (\Phi)(\mathrm{Ca}[\Phi] \supset [\Phi(x) \equiv \Phi(y)].$$

We might then define specific coloration totalities in terms of color identities with appropriate particulars—for example,"$\mathrm{Red}_6(x) =_{\mathrm{def}} \mathrm{E}(x,a)$"—and adopt D_2 as our definition of "Col(F)" with the added stipulation that y is to be a colored object. Theorem T then follows from D_2 and D_1''.

But a number of objections can be raised to defining colors by reference to particulars, only one of which is that if $\mathrm{Red}_6(x) =_{\mathrm{def}} \mathrm{E}(x,a)$, "$\mathrm{Red}_6(a)$" is still (strict) analytic—which, as we saw in Section I, is objectionable not because of ontological commitments but because it does not allow us to say anything of factual content when we predicate red_6 of a. Fortunately, an alternative to D_2 is available which does not draw upon any particulars, and allows individual color words to occur as descriptive primitives. We note first of all that totality concepts are never themselves primitives, but logical constructions based on an empirical relation between properties and objects. When we say that Φ is the totality of x's properties of kind K, we mean not that Φ has an ontological peculiarity which makes it spurn cohabitation with others of its kind, but merely that Φ, which may elsewhere occur within a complex of K's, happens to be all the properties of kind K that x has—or, if Φ is itself a complex, that x happens to have no properties of kind K that do not always accompany Φ. Hence we may define "F is the coloration totality of x" ["*T(F, x)*"] by

$$\mathrm{D}_5 \qquad T(F,x) =_{\mathrm{def}} \mathrm{Ca}(F) \cdot F(x) \cdot (\Phi)(y)[\mathrm{Ca}(\Phi) \cdot \Phi(x) \cdot F(y) \supset \Phi(y)].$$

To say that F is a coloration totality is then merely to say that F is the property necessarily shared by all objects having a common totality of color attributes; namely,

$$\mathrm{D}_6 \qquad \mathrm{Col}(F) =_{\mathrm{def}} (\exists \Phi)(x)[F(x) \equiv T(\Phi, x)],$$

where the biconditional is best understood as intensional equivalence. To prove theorem T, we observe that by $\mathrm{D}_6, \mathrm{Col}(F) \cdot \mathrm{Col}(G) \cdot (F \neq G) \cdot (\exists x)[F(x) \cdot G(x)]$ only when $(\exists \Phi)(\exists \Psi)(\exists x)[\mathrm{T}(\Phi, x)) \cdot \mathrm{T}(\Psi, x)) \cdot (\Phi \neq \Psi)]$. But the latter is self-contradictory, since by D_5, $(\exists x)[\mathrm{T}(\Phi, x) \cdot \mathrm{T}(\Psi, x)]$ entails $(y)[\Phi(y) \equiv \Psi(y)]$, which in an extensional logic implies that $\Phi = \Psi$. (We obtain the same result for an intensional calculus by appropriate substitutions of intensional for material implication in D_5 and D_6.)

It should be noted that the present derivation of theorem T does not restrict its range to uniformly colored objects. The uniform shades of color are but a subclass of the coloration totalities, and a multicolored object can no more be of two different total configurations than a uniformly colored object can (in the second sense) be of two different shades.

IV

To summarize, the gist of my argument has been that while Putnam's explication of color concepts cannot sustain close inspection, partly for technical reasons and partly for its inability to deal with the general case, it would nonetheless seem that his analytic goal is still partly attainable. The present account appears to show that there are two different ways in which a specific shade of color F can be predicated of an object x; one in which we mean that F is *among* the color properties of x, and another in which we mean that F is the *totality* of these. It is only for the latter case that the law of color incompatibility holds. Moreover, while we customarily understand "x is F" in the second sense when x is uniform in color, we employ "x is F" in the *first* sense when x is divisible into differently colored parts and have no simple way at all of speaking about the coloration of an object whose color varies continuously from point to point. It would seem that our conceptual framework for color properties is not entirely in perfect health, and it is to Putnam's credit that his analytic scalpel has bared a hidden abscess.

But while the old enigma, "Nothing can be two colors all over at once" can indeed be shown to be good, old-fashioned analytic to the extent it is true at all, this is by no means the case for all the truths about colors which have so frequently been offered as prima facie examples of the synthetic a priori. In particular, we have here given no support whatsoever to the contention that "Red is a color" is analytic in the strict sense. For we have succeeded here *not* in defining "F is a color" ["Ca(F)"], but only in using "Ca(F)" as a second level *primitive* to define “F is a coloration totality.” So far as the present analysis is concerned, determination of whether any particular attribute is a color is a strictly synthetic judgment.

As a matter of fact, any argument which seeks to prove the analyticity of “Red is a color” by stipulation that colors are defined by color identity to appropriate particulars, with “E(x, y)” [or “I(x, y)”] taken as *primitive* simply begs the issue, for to say that two objects are the same in color is to *presuppose* a way of sorting properties into categories. To say of red circle a that it is indistinguishable in *color* from red square b is to identify the redness, rather than the roundness, of a as its color, and redness, rather than squareness, as the color of b.

Putnam has reiterated that he knows of no philosopher who has taken “Red is a color,” or “Anything which is red is colored,” as apodictic in any interesting sense. At the risk of exposing my philosophical naiveté, I must confess that I have long considered these to be the quintessence of synthetic a priori truth. I would hasten to agree, of course, that the synthetic a priori here is truth *ex vi terminorum*, and indeed, I would question whether there is a case of the former which is not also a case of the latter. I would also question, however, whether “true by virtue of the meanings of the terms” is properly equated to “true as a consequence of the rules of the language.” But this is another story, and a long, involved one at that.

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