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New Dimensions of Confirmation Theory

When Hempel's "paradox of confirmation" is developed within the confines of conditional probability theory, it becomes apparent that two seemingly equivalent generalities ("laws") can have exactly the same class of observational refuters even when their respective classes of confirming observations are importantly distinct. Generalities which have the inductive supports we commonsensically construe them to have, however, must incorporate quasi-logical operators or connectives which cannot be defined truth-functionally. The origins and applications of these "modalic" concepts appear to be intimately linked with a number of basic conundrums in the philosophy of science, such as causation and the nature of explanation.

While Hempel's "paradox of confirmation" has enjoyed considerable attention of late,[1] the deepest layers of this puzzle still remain unprobed. As we shall see, once these are reached, not only the paradox's resolution but also why it has seemed puzzling become obvious, even trivial. But not at all trivial are the powerful new concepts and problems disclosed by this penetration concerning the structure of belief, the basis of explanation, and the inferential force of propositional connectives.

In this paper, I shall attempt to light a fuse under certain explosive implications which appear in our *de facto* inference habits when these are reconstructed by confirmation theory within the confines of the probability calculus. Unlike classical logic, confirmation theory recognizes that rational inference includes not merely formal deduction but also adjustments in the credibility of uncertain hypotheses in light of relevant albeit inconclusive evidence. Insomuch as credibility—i.e. appropriate belief—is a matter of degree, technical development of confirmation theory requires some *measure* of the credibility of a proposition p given evidence e. Virtually any such measure which intuitively corresponds to a "rational" pattern of belief satisfies the conditional probability axioms—in fact, under modestly ideal circumstances, this is true even when the credibility measure is no more than an ordinal scale reflecting the "is-more-credible-than" relation (cf. Savage, 1954). Accordingly, I shall take *the probability (credibility) that p is the case, given that e is the case*—abbreviated $\Pr(p \mid e)$—to be a basic concept of confirmation theory and will assume in what follows that all theorems of the probability calculus hold for

[1] See Hempel (1966), Scheffler (1963), Schlesinger (1965), and numerous other references cited therein.

it. The unconditional (i.e. "prior") probability, $\Pr(p)$, of a proposition p may be construed to be p's conditional probability relative to certain background knowledge of a very general sort, the nature of which need not here concern us. Evidence e then confirms or disconfirms proposition p according to whether $\Pr(p \mid e)$ is respectively larger or smaller than $\Pr(p)$. (More generally, e confirms or disconfirms p relative to background information b according to whether $\Pr(p \mid e \cdot b)$ is larger or smaller than $\Pr(p \mid b)$.)

I shall also make one further assumption about probabilities which, though not strictly necessary to my fundamental point, greatly expedites its development. Whereas confirmation theory interprets "probability" as a measure of credibility relationships among *propositions*, it is well known that as the term is used in the natural sciences, "probability" refers primarily to certain poorly understood but well-axiomatized relations between *properties* (cf. Carnap, 1950, p. 35; Rozeboom, 1961). That is, probabilities in this second sense are something described by expressions of form '$\Pr(Q \mid P) = r$' wherein 'P' and 'Q' are predicates rather than sentences, and where the numerical value of $pr(Q \mid P)$ is approximately equal to the relative frequency of Q-type things among things which have property P. (Present notation will use 'pr' to differentiate this kind of probability from the 'Pr' of propositional probability.) While the precise relation between these two kinds of probabilities is still controversial, there can surely be little doubt that an important and intimate connection does exist. One intuitively plausible conjecture which would do nicely for present purposes is that if h is a generality which entails that $pr(Q \mid P) = r$, then for an arbitrary object a the credibility of Qa given h and Pa is also equal to r. Unfortunately, however, this principle is demonstrably not universal,[2] and its boundary restrictions are still too blurred to risk grounding a serious argument upon it. Instead, I shall postulate merely that if generality h makes no difference for the probability of property Q relative to property P, then neither does it affect the credibility that an arbitrary object a has Q given that a is a P. That is, so long as a is not singled out for special mention in h,

$$(r)\{\Pr[pr(Q \mid P) = r \mid h] = \Pr[pr(Q \mid P) = r]\} \supset \Pr(Qa \mid Pa \cdot h) = \Pr(Qa \mid Pa),$$

a special case of which (when P is vacuous) is that if h does not affect the property probability of Q then neither is h relevant to the credibility of o's being Q. Although the logistical complexity of this assumption is somewhat disconcerting on first glance, it is scarcely conceivable that any coherent theory of the $\Pr/pr$ interface would not entail some such irrelevance principle. Thus while equations (11)–(16), below, are grounded partly on intuition, it is an intuition which has

[2]Specifically, Miller (1966) has derived paradox from the special case, $\Pr[Qa \mid pr(Q) = r] = r$, of this principle, and although Miller's argument exploits a correctable technical ambiguity, further difficulties lie deep within this principle's substantive content (see Rozeboom, 1969).

every right to be taken seriously until strong arguments to the contrary are forthcoming.

I

For any two propositions p and q, it is elementary to show (cf. fn. 8, below) that if p entails q then q as evidence confirms p provided only that neither p, $\sim p$, nor q were certain to begin with.[3] However, this theorem has only limited relevance for the confirmation of scientific generalities insomuch as these are usually conditionals which fail to imply the categorical data by which they are confirmed. Consider, for example, the hypothesis that

(1) All As are Bs

i.e.

(la) If anything is an A, then it is also a B,

which may be formalized as

(2) $(x)(Ax \rightarrow Bx)$

wherein '$\rightarrow$' abbreviates 'If ..., then ...' in whatever sense is intended by (la). How might this hypothesis be confirmed by observing the properties of particular objects ? Common sense cries out that data of the form

(3) $Aa \cdot Ba$

—i.e. evidence that some particular A is also a B—confirm (2). But all which can be *deduced* about object a from generality (2) is that

(4) $Aa \rightarrow Ba$,

and even if (3), too, implies (4), it is important to appreciate that evidence which verifies an uncertain consequent of a proposition p need not confirm p itself. That is, if p and e both entail q while $0 < \Pr(p) < 1$ and $\Pr(q) < 1$, then $\Pr(p \mid q) > \Pr(p)$ but not necessarily $\Pr(p \mid e) > Pr(p)$. Thus our intuition that (3) confirms (2) cannot be justified merely by appeal to the principle that evidence supports

[3] Confirmation of a hypothesis by verification of its consequent need not be inferentially *significant*, however. For discussion of this point together with demonstration that the "hypothetico-deductive" view of scientific inference is generically vacuous, see Rozeboom, 1970.

those hypotheses which entail it; for insomuch as (3) includes datum Aa, it is stronger than any consequence of (2).

Just as we feel that determining whether an object known to be an A is or is not a B respectively confirms or refutes hypothesis (2), so does it also seem that learning the B-state of something which is not an A, i.e. an observation of form

(5) $\quad \sim Aa \cdot Ba$

or

(6) $\quad \sim Aa \cdot \sim Ba,$

has no relevance to the credibility of (2). But by contraposition, (1) appears logically equivalent to 'All non-Bs are non-As,' i.e.

(7) $\quad (x)(\sim Bx \rightarrow \sim Ax),$

which by the same intuition as before is confirmed by (6) but not by (3) or (5). Moreover, another prima facie equivalent to (1), namely 'Everything is either a non-A or a B', or

(8) $\quad (x)(\sim Ax \vee Bx),$

seems by virtue of its symmetry in $\sim A$ and B to be confirmed by either (3), (5), or (6). This is the "paradox of confirmation," namely, the incompatibility of our commonsense convictions (i) that (3) but not (5) or (6) confirms (2), (ii) that (6) but not (3) or (5) confirms (7), (iii) that (3), (5), and (6) all confirm (8), and (iv) that (2), (7), and (8) are all logically equivalent. (That (i)-(iv) are incompatible follows from the elementary theorem that for any three propositions p, q, and r such that p and q are logically equivalent, r confirms p iff r confirms q.)

While the assumed equivalence of 'All As are Bs,' 'All non-Bs are non-As,' and 'Everything is either a non-A or a B' is not above suspicion, the most pressing challenge to confirmation theory raised by the paradox of confirmation is simply that of determining, by arguments more convincing than naked intuition, just what degree of confirmation *is*, in fact, given to a generalized conditional by a particular observation which does not refute it. In our present case, if h is hypothesis (2) while d is datum (3), (5), or (6), we need to deduce how $\Pr(h \mid d)$ compares with $\Pr(h)$. But it is a fundamental theorem of confirmation theory that

$$(9^4) \qquad \frac{\Pr(h \mid d)}{\Pr(h)} = \frac{\Pr(d \mid h)}{\Pr(d)}.$$

Hence d confirms h iff h confirms d, and our original query about the confirmation of hypothesis (2) by observation (3), (5), or (6) is answerable if and only if we are able to determine how the respective probabilities of (3), (5), and (6), given (2), compare with their prior probabilities.

Our problem, therefore, may be recast as follows: To confirm or disconfirm generality (2), we propose to observe the status of an arbitrary object a with regard to properties A and B, and to adjust our degree of belief in this law in accord with the shift from its prior probability to its probability given the observation. The four possibilities for our to-be-acquired datum are, of course,

$$\begin{array}{rr} Aa \cdot Ba, & Aa \cdot \sim Ba, \\ \sim Aa \cdot Ba, & \sim Aa \cdot \sim Ba, \end{array}$$

while as just noted, we can interpret each of these in regard to the credibility of (2) only if we know what the law, in turn, implies about the probability of this particular observation. The probability of $Aa \cdot \sim Ba$ given (2) is clearly zero—which is why this observation would, conversely, drop the posterior probability of (2) to vanishing. But what probabilities does (2) confer upon the three datum alternatives with which it is compatible? One is tempted to argue that (2) merely disclaims the existence of any A which is not also a B and should hence say nothing about the respective probabilities of being both A and B, both non-A and B, or both non-A and non-B. But this cannot be so, for insomuch as $\Pr(Aa \cdot Ba \mid (2)) + \Pr(Aa \cdot \sim Ba \mid (2)) + \Pr(\sim Aa \cdot \sim Ba \mid (2)) = 1 = \Pr(Aa \cdot Ba) + \Pr(Aa \cdot \sim Ba) + \Pr(\sim Aa \cdot Ba) + \Pr(\sim Aa \cdot \sim Ba)$ while $\Pr(Aa \cdot \sim Ba \mid (2)) = 0$, we have

$$\begin{aligned} (10) \qquad & \{\Pr(Aa \cdot Ba \mid (2)) - \Pr(Aa \cdot Ba)\} \\ & \quad +\{\Pr(\sim Aa \cdot Ba \mid (2)) - \Pr(\sim Aa \cdot Ba)\} \\ & \quad +\{\Pr(\sim Aa \cdot \sim Ba \mid (2)) - \Pr(\sim Aa \cdot \sim Ba)\} \\ & = \Pr(Aa \cdot \sim Ba); \end{aligned}$$

whence if $\Pr(Aa \cdot \sim B) \neq 0$, at least one of the three terms in braces on the left-hand side of (10) must be positive. Thus so long as a joint occurrence of A and $\sim B$ is not impossible at the outset, at least one of the observational possibilities, $Aa \cdot Ba$, $\sim Aa \cdot Ba$, or $\sim Aa \cdot \sim Ba$, must confirm hypothesis $(x)(Ax \rightarrow Bx)$. More generally, *any hypothesis which entails that certain combinations of properties have zero probability of occurrence also requires a compensatory alteration of probabilities over those property-combinations which the hypothesis does not exclude.* Accordingly, full specification of a statement's meaning must clarify not

[4]Proof: Since $\Pr(p \mid q) = \Pr(p \cdot q)/\Pr(q)$ for any propositions p and q, each side of (9) equals $\Pr(h \cdot d)/[\Pr(h) \times \Pr(d)]$

merely its deductive force but its probabilistic implications as well. In particular, it is premature to assume that two generalities which agree in what observational possibilities they exclude necessarily confer the same probabilities upon the various observations with which they are compatible.

II

The supposedly paradigmatic examples of data and theory which have become traditional in philosophic reconstructions of the scientific enterprise are, in logical complexity, usually but pale shadows of the real thing. Convenient as these simplifications may be for many analytic purposes, they also obliterate details and subtleties which often carry the main thrust of technical science. This is especially true of typical philosophic accounts of scientific "laws," for sentence-schema 'All ϕs are ψs' is far too primitive for accurate description of natural regularities. Instead, modern science employs a variety of regularity concepts falling under the general rubric 'Variables $\mathbf{X}$ and $\mathbf{Y}$ are related under boundary conditions B in fashion r'.[5] (A "variable" in scientific parlance is a set of properties which are mutually exclusive and jointly exhaustive over a given domain of objects, while a variable's "value" for a member of its domain is that one property in the relevant set which is true of that object. For details, see Rozeboom, 1966.) Technical measures of bivariate (and, more generally, multivariate) relatedness are of two fundamentally distinct kinds: symmetric *vs.* asymmetric. Symmetric measures such as correlation coefficients describe some aspect of how variables $\mathbf{X}$ and $\mathbf{Y}$ *co-vary* under conditions B—i.e. what combinations of values on $\mathbf{X}$ and $\mathbf{Y}$ tend to be more prevalent than others. The complete symmetric relationship between $\mathbf{X}$ and $\mathbf{Y}$ in B is given by the "joint distribution" of $\mathbf{X}$ and $\mathbf{Y}$ in B, namely, the frequency, probability, or probability density of each combination of a value X_i of $\mathbf{X}$ with a value Yj of $\mathbf{Y}$ among objects satisfying conditions B. Asymmetric relational measures such as regression coefficients, on the other hand, characterize how variable $\mathbf{Y}$ *depends upon* variable $\mathbf{X}$ (or conversely) under conditions B, while the complete statistical dependency of $\mathbf{Y}$ upon $\mathbf{X}$ in B is given by the function which maps each value X_i of $\mathbf{X}$ into the contingent distribution of $\mathbf{Y}$, given X_i, in B. (The "contingent distribution" of $\mathbf{Y}$, given X_i, in B is the distribution of $\mathbf{Y}$—i.e. the frequency, probability, or probability density of each value Y_j of $\mathbf{Y}$—under the boundary conditions of having value X_i of $\mathbf{X}$ as well as satisfying B.) Mathematically, any contingent distribution of $\mathbf{Y}$ given X_i in B is equally tolerant of any arbitrary frequency, probability, or probability density in B for X_i, so

[5] Even this is insufficient to cover all forms of lawful relationships dealt with by contemporary science. Over and above the fact that research on the simultaneous relatedness of an arbitrarily large number of variables has been burgeoning over the past century, new methodologies have recently emerged for detection and analysis of data patterns to which the more traditional multivariate concepts do not apply (see Rozeboom, 1966, pp. 197–214).

the dependence of $\mathbf{Y}$ upon $\mathbf{X}$ in B is not ordinarily thought to reveal anything about how the independent variable $\mathbf{X}$ is itself distributed in B. More generally, there is common agreement in research practice that the distribution of one or more variables under a given set of boundary conditions should be regarded as a statistical datum which tells nothing about the prevalence of these boundary conditions themselves, nor about the distribution of these variables under alternative boundary conditions, except insofar as we have acquired some higher-level empirical generalizations which support inference from one statistic of this sort to another.

The contrast between symmetric and asymmetric measures of bivariate relatedness in large measure reflects the empirical methods by which natural regularities are identified. In correlational research, one obtains a hopefully random sample of joint observations on variables $\mathbf{X}$ and $\mathbf{Y}$ under boundary conditions B and interprets these as a sample-frequency approximation to the joint distribution of these variables in B. Whereas in dependency studies (often referred to as "experimental" method in contrast to the "field observations" of correlational technique), a chosen number of $\mathbf{Y}$-values are observed under conditions B at each of certain experimenter-selected or experimenter-induced values of $\mathbf{X}$, and the relative frequencies of observed $\mathbf{Y}$-values at each observed $\mathbf{X}$-value X_i taken as a sampling approximation to the underlying contingent distribution of $\mathbf{Y}$ given X_i in B. Insomuch as the latter method imposes a distribution on $\mathbf{X}$ in the observed sample, it can yield no information about $\mathbf{X}$'s underlying distribution in B. Neither does it intrinsically reveal anything about the contingent distributions of $\mathbf{Y}$ in B at unobserved values of $\mathbf{X}$, though in practice the function mapping observed $\mathbf{X}$-values into contingent $\mathbf{Y}$-distributions is often sufficiently regular that we feel reasonably confident in generalizing it to unobserved $\mathbf{X}$-values as well.

To illustrate these habits of statistical thought, consider the inductive implications of certain hypothetical information which might be obtained about the relationship within the population of humans between two dichotomous variables-say Sex and Handedness, the values of which are ⟨being male, being female⟩ and ⟨being right-handed, being left-handed⟩, respectively. Since the distribution of a dichotomous (i.e. two-valued) variable is completely characterized by the prevalence of one of its values, the statistical dependence of Handedness upon Sex in humans may be described by stating for each sex the probability that a person of that sex is right-handed. Suppose, now, that you are informed that 86% of male humans are right-handed. This gives you the contingent distribution of human Handedness at one value—male—of the Sex variable, but what does it imply about (a) the contingent distribution of Handedness in human females, or about (b) the distribution in humans of Sex itself? With respect to (a), inferring that probably about 86% of female humans, too, are right-handed is justified only to the extent that you are willing likewise to infer that Handedness is statistically independent of Sex among humans—an unreasonable assumption to make in view of the profound difference you know Sex to make for so many other physical and behavioral characteristics. More precisely, being given the incidence of right-handedness in human males converts your prior distribution of probabilities over the various possible dependencies Handedness might have upon Sex in humans into a posterior distribution of probabilities for the distribution of Handedness in human females, which distribution of distributions then collapses into a posterior probability of female right-handedness given the handedness data for males. Just what this posterior probability may be depends upon your background knowledge prior to learning the male-handedness statistic, but there is no obvious reason why it should diverge appreciably from your prior expectation of right-handedness in human females—certainly the male datum gives you no *primary* inductive grounds on which to revise your prior beliefs about females. And as for (b), there seems to be no reason at all for inferring anything about the prevalence of masculinity in humans from information that *within* the class of male humans, whatever its incidence, 86% are right-handed. Roughly speaking, then, we may say that the primary, or conservative, interpretation of evidence comprising the contingent distribution of one variable $\mathbf{Y}$ at a fixed value of another variable $\mathbf{X}$ under boundary conditions B is to leave unaltered one's beliefs both about the distribution of $\mathbf{X}$ in B and about the contingent distributions of $\mathbf{Y}$ in B at other values of $\mathbf{X}$.

On the other hand, if you are informed only that 41% of humans are both male and right-handed, you have learned *part* of the joint distribution of Sex and Handedness in humans, but what does this tell you about the remainder of the distribution? Certainly it reveals nothing about how Sex and Handedness co-vary under these boundary conditions unless you are also given the marginal distributions of Sex and Handedness, even though it does make some difference for what the latter are likely to be, and the conservative interpretation is for your beliefs about the joint distribution of Sex and Handedness contingent upon a person's being *not* both male and right-handed to remain unchanged from what they were before you learned the incidence of right-handed males among humans.

The fundamental point to be made here is simply that the contingent distribution of a variable $\mathbf{Y}$ at a given value X_i of a variable X under boundary conditions B is a very different datum from the joint frequency of a particular combination of values on $\mathbf{X}$ and $\mathbf{Y}$ in B, and that by the same token both of these differ significantly from the contingent distribution of $\mathbf{X}$ in B at a given value Y_j of $\mathbf{Y}$. These differences are deeply rooted in technical conceptions of bivariate relatedness, and while the respective implications of these statistics for other aspects of the total bivariate structure from which they are an abstraction depend importantly upon the details of whatever additional knowledge is also available, the *primary* induction in each case is that if d asserts only that some distributional property holds within a subclass B_i of B while all that p claims is either (1) that certain things are true of some distribution within a subclass of B alternative to B_i or (2) that B_i and its alternatives have a certain distribution in B, then $\Pr(p \mid d) = \Pr(p)$.

III

When ordinary-language generality (1) is inspected from the vantage point of distribution theory, the grammatical style in which it predicates something of the class of objects having property A strongly urges that we interpret it as specifying the contingent distribution of the variable $\mathbf{B} =_{\text{def}} \langle B, \sim B\rangle$ among just those objects having value A of the variable $A =_{\text{def}} \langle A, \sim A\rangle$.[6] That is, if D is the domain of the quantifier in (2), 'All As are Bs' essentially says that the contingent distribution of variable $\mathbf{B}$ in D at value A of variable $\mathbf{A}$ is such that B occurs with 100% incidence. By the same grammatical principle, we are urged to interpret 'All non-Bs, are non-As' as basically a statement about the contingent distribution of variable $\mathbf{A}$ at value $\sim B$ of variable $\mathbf{B}$. And finally, the natural impact of 'Everything is either a non-A or a B' is symmetric in 'non-A' and 'B' with neither providing a grammatical subject for the sentence, is that of a partial description of the joint distribution of variables $\mathbf{A}$ and $\mathbf{B}$ in D, namely, as a claim that joint occurrence of A and non-B has zero incidence in D. But then it is *not* correct to regard propositions (2), (7), and (8) as logical equivalents. (Note that even in this highly special case with dichotomous variables and extreme incidence rates, $pr(B \mid A) = 1$ cannot be deduced from $pr(\sim A \vee B) = 1$ or from $pr(\sim A \mid \sim B) = 1$ unless it is also given that $pr(A) \neq 0$.) And if (2), (7), and (8) do not say precisely the same thing, then neither is it paradoxical that they have somewhat different confirmational patterns.

To be explicit, consider what the primary inductive interpretations of (2), (7) and (8) respectively imply, by way of my introductory assumption relating the probabilities of propositions to the probabilities of properties, about their confirmation by the joint values on A and B of a to-be-observed object a. Placing an 'i' over the "given"-bar in those probabilities which remain unaltered from their prior values under primary induction from the assumed generality and also over the equalities which follow by means of the latter, while '1' or '0' over the bar denotes a probability implied by the assumed generality to be unity or zero, respectively, we have

[6] In scientific practice, A and B would most likely be regarded as values of multi-valued variables—e.g. 'All rubies are red,' wherein *being a ruby* is best construed as one of many alternative gemstone types while *being red* is one of many alternative colorations. For heuristic purposes, however, any variable $\mathbf{X}$ of which X_i is a value can always be collapsed into a dichotomous variable $\mathbf{X}^*$ whose two values are, respectively, X_i and the disjunction of all $\mathbf{X}$-values other than X_i.

(11[7])
$$
\begin{aligned}
\Pr(Aa \cdot Ba \mid (2)) &= \Pr(Aa \overset{i}{\mid} (2)) \times \Pr(Ba \overset{1}{\mid} Aa \cdot (2)) \overset{i}{=} \Pr(Aa),\\
\Pr(Aa \cdot \sim Ba \mid (2)) &= \Pr(Aa \overset{i}{\mid} (2)) \times \Pr(\sim Ba \overset{0}{\mid} Aa \cdot (2)) = 0,\\
\Pr(\sim Aa \cdot Ba \mid (2)) &= \Pr(\sim Aa \overset{i}{\mid} (2)) \times \Pr(Ba \overset{i}{\mid} \sim Aa \cdot (2))\\
&\overset{i}{=} \Pr(\sim Aa) \times \Pr(Ba \mid \sim Aa)\\
&= \Pr(\sim Aa \cdot Ba),\\
\Pr(\sim Aa \cdot \sim Ba \mid (2)) &= \Pr(\sim Aa \overset{i}{\mid} (d)) \times \Pr(\sim Ba \overset{i}{\mid} \sim Aa \cdot (2))\\
&\overset{i}{=} \Pr(\sim Aa) \times \Pr(\sim Ba \mid \sim Aa)\\
&= \Pr(\sim Aa \cdot \sim Ba)
\end{aligned}
$$

for the primary inductive implications of ‘All As are Bs’, whereas those of ‘All non-Bs are non-As’ for these same datum possibilities are

(12)
$$
\begin{aligned}
\Pr(Aa \cdot Ba \mid (7)) &= \Pr(Ba \overset{i}{\mid} (7)) \times \Pr(Aa \overset{i}{\mid} Ba \cdot (7))\\
&\overset{i}{=} \Pr(Ba) \times \Pr(Aa \mid Ba)\\
&= \Pr(Aa \cdot Ba)\\
\Pr(Aa \cdot \sim Ba \mid (7)) &= \Pr(\sim Ba \overset{i}{\mid} (7)) \times \Pr(Aa \overset{0}{\mid} \sim Ba \cdot (7)) = 0,\\
\Pr(Aa \cdot Ba \mid (7)) &= \Pr(Ba \overset{i}{\mid} (7)) \times \Pr(\sim Aa \overset{i}{\mid} Ba \cdot (7))\\
&\overset{i}{=} \Pr(Ba) \times \Pr(\sim Aa \mid Ba)\\
&= \Pr(\sim Aa \cdot Ba),\\
\Pr(\sim Aa \cdot \sim Ba(7)) &= \Pr(\sim Ba \overset{i}{\mid} (7)) \times \Pr(\sim Aa \overset{1}{\mid} \sim Ba \cdot (7))\\
&= \Pr(\sim Ba).
\end{aligned}
$$

Finally, under the partial-joint-distributional interpretation of ‘Everything is either non-A or B’,

(13[8])
$$
\begin{aligned}
\Pr(Aa \cdot Ba \mid (8)) &= \Pr(\sim Aa \vee Ba \overset{1}{\mid} (8)) \times \Pr(Aa \cdot Ba \overset{i}{\mid} (\sim Aa \vee Ba) \cdot (8))\\
&\overset{i}{=} \Pr(Aa \cdot Ba \mid \sim Aa \vee Ba)\\
&= \Pr(Aa \cdot Ba)\ /\ \Pr(\sim Aa \mid Ba),\\
\Pr(Aa \cdot \sim Ba \overset{0}{\mid} (8)) &= 0,\\
\Pr(\sim Aa \cdot Ba \mid (8)) &= \Pr(\sim Aa \vee Ba \overset{1}{\mid} (8)) \times \Pr(\sim Aa \cdot Ba \overset{i}{\mid} (\sim Aa \vee Ba) \cdot (8))
\end{aligned}
$$

[7] To clarify the proofs for (11)–(13), it will suffice to show the derivation for $\Pr(Aa \cdot Ba \mid (2))$. From the probability calculus we have $\Pr(Aa \cdot Ba \mid (2)) = \Pr(Aa \mid (2)) \times \Pr(Ba \mid Aa \cdot (2))$. But $\Pr(Ba \mid Aa \cdot (2)) = 1$ since Aa and (2) jointly entail Ba; while since by primary induction (2) makes no difference for the probability of property A, the assumed $\Pr/pr$ irrelevance principle implies that $\Pr(Aa \mid (2)) = \Pr(Aa)$. Hence $\Pr((Aa \cdot Ba \mid (2)) = \Pr(Aa) \times 1 = \Pr(Aa)$.

$$
\begin{aligned}
&\stackrel{i}{=} \Pr(\sim Aa \cdot Ba \mid \sim Aa \vee Ba)\\
&= \Pr(\sim Aa\cdot \sim Ba)/\Pr(\sim Aa \vee Ba),\\
\Pr(\sim Aa \cdot \sim Ba \mid (8)) &= \Pr(\sim Aa \vee Ba \stackrel{1}{\mid} (8)) \times \Pr(\sim Aa \cdot \sim Ba \stackrel{i}{\mid} (\sim Aa \vee Ba) \cdot (8))\\
&\stackrel{i}{=} \Pr(\sim Aa \cdot \sim Ba \mid \sim Aa \vee Ba)\\
&= \Pr(\sim Aa \cdot \sim Ba) \;/\; \Pr(\sim Aa \vee Ba).
\end{aligned}
$$

Hence using principle (9), the confirmation ratios for these three versions of the generality under each possible observation of joint values on **A** and **B** are

(14)
$$
\begin{aligned}
\frac{\Pr((2) \mid Aa \cdot Ba)}{\Pr((2))} &= \frac{1}{\Pr(Aa \cdot Ba)},\\
\frac{\Pr((2) \mid Aa \cdot \sim Ba)}{\Pr((2))} &= 0,\\
\frac{\Pr((2) \mid \sim Aa \cdot Ba)}{\Pr((2)} &= 1,\\
\frac{\Pr((2) \mid \sim Aa \cdot \sim Ba)}{\Pr((2))} &= 1,
\end{aligned}
$$

for ‘All As are Bs’;

(15)
$$
\begin{aligned}
\frac{\Pr((7) \mid Aa \cdot Ba)}{\Pr((7))} &= 1,\\
\frac{\Pr((7) \mid Aa \cdot \sim Ba)}{\Pr((7))} &= 0,\\
\frac{\Pr((7) \mid \sim Aa \cdot Ba)}{\Pr((7)} &= 1,\\
\frac{\Pr((7) \mid \sim Aa \cdot \sim Ba)}{\Pr((7))} &= \frac{1}{\Pr(\sim Aa \cdot \sim Ba)},
\end{aligned}
$$

for ‘All non-Bs are non-As; and

(16)
$$
\begin{aligned}
\frac{\Pr((8) \mid Aa \cdot Ba)}{\Pr((8))} &= \frac{1}{\Pr(\sim Aa \vee Ba)},\\
\frac{\Pr((8) \mid Aa \cdot \sim Ba)}{\Pr((8))} &= 0,\\
\frac{\Pr((8) \mid \sim Aa \cdot Ba)}{\Pr((8)} &= \frac{1}{\Pr(\sim Aa \vee Ba)},\\
\frac{\Pr((8) \mid \sim Aa \cdot \sim Ba)}{\Pr((8))} &= \frac{1}{\Pr(\sim Aa \vee Ba)},
\end{aligned}
$$

[8] Cases $Aa \cdot Ba, \sim Aa \cdot Ba$, and $Aa \cdot \sim Ba$ here utilize principles (a) that if p logically entails q, then for any additional proposition r, $\Pr(p \mid r) = \Pr(q \mid r) \times \Pr(p \mid q \cdot r) + \Pr(\sim q \mid r) \times \Pr(p \mid \sim q \cdot r) = \Pr(q \mid r) \times \Pr(p \mid q \cdot r)$, and (b) that when p entails q, $\Pr(p \mid q) = \Pr(p)/\Pr(q)$, which follows from (a) by letting r be tautological.

for 'Everything is either a non-A or a B'. Thus under primary induction, observation $Aa \cdot Ba$ would confirm (2) but not (7), observation $\sim Aa \cdot \sim Ba$ would confirm (7) but not (2), while any one of the observations $Aa \cdot Ba$, $\sim Aa \cdot Ba$, or $\sim Aa \cdot \sim Ba$ would confirm (8)—exactly as intuition says should be so.

IV

So much for the "paradox of confirmation" in its classical formulation. Its resolution consists in showing that the three generalities, 'All As are Bs,' 'All non-Bs are non-As,' and 'Everything is either a non-A or a B,' are not commonsensically understood to mean exactly the same thing, as manifested by their failure to effect the same pattern of adjustment in the credibilities we attach to the various possible observations with which they are compatible. But this only confronts us with a deeper problem: What must be the character of the connective in (2) if the probabilistic conclusions we typically draw from it are to be a legitimate inference from 'If . . . , then . . .' statements of this sort?

Since the point I now wish to make is more directly accessible through analysis of nonqualified conditionals than through conditional generalities, I shall assume that there is a sense—in fact, probably the most common sense—of 'If . . . , then . . .' which is ascribed directly to complete propositions rather than to propositional functions as in (2), but which has essentially the same pattern of primary-inductive implications as does the connective in (2). (One could, in fact, argue that this is inherent in (11) on the grounds that $(x)(Ax \to Bx)$ entails $Aa \to Ba$ by instantiation of the quantifier, while it is the law's particularization for object a which mediates the law's probabilistic implications about a's properties. However, insomuch as this argument is not impeccable, I shall bypass it.) Specifically, I presume that when '$p \to q$' expresses this sort of conditional coupling between two propositions p and q, $p \to q$ makes a difference for the probability of q given also p, but not for the probability of p. Certainly in everyday linguistic practice we often *intend* our conditional assertions to have this sort of force. Thus when I speculate, "If it freezes tonight, my car won't start in the morning," or "If Jim's hole card isn't a queen, then I'll win this pot," I do not consider the contingency I am entertaining to make the slightest difference for whether it *will* freeze tonight or whether Jim's hole card *is* a queen. *But if in general* $\Pr(p \mid p \to q) = Pr(p)$, *then the '$\to$'-connective cannot be explicated as material implication.* For insomuch as $\sim p$ entails $p \to q$ by the definitional equivalence of $p \supset q$ with $\sim (p \cdot \sim q)$ we have $\Pr[\sim p \cdot (p \supset q)] = \Pr(\sim p)$; hence

$$\begin{aligned}\Pr(p \mid p \supset q) &= 1 - \Pr(\sim p \mid p \supset q) = 1 - \frac{\Pr[\sim p \cdot (p \supset q)]}{\Pr(p \supset q)} \\ &= 1 - \frac{\Pr(\sim p)}{\Pr(p \supset q)} = \Pr(p) + \Pr(\sim p) - \frac{\Pr(\sim p)}{\Pr(p \supset q)} \\ &= \Pr(p) - \Pr(\sim p)\left[\frac{1 - \Pr(p \supset q)}{\Pr(p \supset q)}\right],\end{aligned}$$

or expressed as a confirmation ratio,

$$\text{(17)} \qquad \frac{\Pr(p \mid p \supset q)}{\Pr(p)} = 1 - \frac{\Pr(\sim p)}{\Pr(p)} \times \frac{\Pr[\sim (p \supset q)]}{\Pr(p \supset q)}.$$

Thus unless the prior probability of p or of $p \rightarrow q$ is unity, the probability of p given $p \supset q$ is necessarily *less* than the prior probability of p—whence if $\Pr(p \mid p \rightarrow q) = \Pr(p)$ even while p and $p \rightarrow q$ have some prior uncertainty, $p \rightarrow q$ cannot be logically equivalent to $p \supset q$. The former entails the latter, but not conversely.

It is not possible to be so definite about the unacceptability of material implication for explicating the conditional in generalities such as (2), for the probability calculus does not in itself authorize any deductive conclusions about how the probability of property A (or the proposition-probability that an arbitrary object a is an A) given $(x)(Ax \supset Bx)$ differs from the unconditional probability of A. Since $(x)(Ax \supset Bx)$ is logically equivalent to (8), it can be deduced from (13) that

$$\text{(18)} \qquad \frac{\Pr[Aa \mid (x)(Ax \supset Bx)]}{\Pr(Aa)} = 1 - \frac{\Pr(\sim Aa)}{\Pr(Aa)} \times \frac{\Pr[\sim (Aa \supset Ba)]}{\Pr(Aa \supset Ba)};$$

however, it is not absolutely certain that probabilistic implications (13) really are inherent in the meaning of (8). Even so, it can in any event be shown that

$$\text{(19)} \qquad \frac{\Pr[Aa \mid (x)(Ax \supset bx) \cdot h^*]}{\Pr(Aa \mid h^*)} = 1 - \frac{\Pr(\sim Aa \mid h^*)}{\Pr(Aa \mid h^*)} \times \frac{\Pr[\sim (Aa \supset Ba) \mid h^*]}{\Pr(Aa \supset Ba \mid h^*)},$$

where

$$h^* =_{\text{def}} (X)[(x \neq a) \cdot Ax \supset Bx],$$

so there is at least a mathematical tendency for $(x)(Ax \supset Bx)$ to imply a decreased probability of Aa even if the auxiliary hypothesis that object a is at most the only exception to $(x)(Ax \supset Bx)$ is needed to make this tendency explicit. It appears, then, that replacement of '$\rightarrow$ in (2) by '$\supset$' would be in fundamental violation of the probabilistic inferences which are usually drawn from conditional-distribution information in real life, even when the purity of primary induction as in (14) is modulated by secondary inductions. Either these *de facto* inferences are badly misguided or there is a truly *conditional* sense of 'If ... , then ... ' which cannot be truth-functionally reduced to 'Either not-... or ...'.

V

But if 'All As are Bs' and 'If p, then q' generally say more than that $(x)(Ax \supset Bx)$ and that $p \supset q$, respectively, what then *is* the connective's meaning in such cases? I am not at all sure that I can answer this satisfactorily, partly because there may well exist a whole family of conditionals stronger than '$\supset$' and partly because these may be conceptually primitive, amenable to explication only through exhibiting their logical grammar and contexts of usage. But I can at least point out a fragment of the answer, and a most provocative fragment it is.

The origin of the conditionality concept (concepts ?) in question—neither quite logical nor quite descriptive—very likely resides in the responsibility ordering of judgments. A person's beliefs at any given moment are not an accidental aggregate of disconnected opinions, but partake of an inferential cohesiveness by which some of these beliefs are, for that person, the psychological basis of the rest. A person believes proposition p to a certain degree *because* he believes proposition q, and believes q in turn because he jointly believes propositions r and s, etc. Even when two propositions are logically equivalent, their positions within a particular belief structure will generally be asymmetric in that the degree of conviction invested in the one derives from the conviction sustained by the other. Now psychologically, it is but a short step from believing p because of believing q to believing that p is the case because q is the case. Whether or not this is the notion's sole origin, it is in any event a fundamental fact of human reason that many of our judgments are explicitly of form 'p because q', while a great many more such beliefs are latent in the explanations we are willing to accept. It will surely be agreed, for example, that the conjunctive fact, say, that John and Jim are both married, is due jointly to John's being married and to Jim's being married, while my birthday is in February because I was born on Feb. 29, 1928. The direction of *responsibility* in such cases may or may not coincide with the direction of entailment: $p \cdot q$ entails p which in turn entails $p \vee q$, yet the truth of both $p \cdot q$ and $p \vee q$ is derivative from the truth of p—i.e. p's being the case is (in part) *why* $p \cdot q$ and $p \vee q$ are the case. More generally, it is intuitively evident that the truth of any molecular proposition $F(p_1, \cdots, p_n)$ which is a truth-function of propositions $p_1, \cdots, p_n$ depends upon (derives from, is due to, originates in, has as its source) the truth-states of $p_1, \cdots, p_n$ but not conversely. Note moreover that while the apparent direction of propositional responsibility is ordinarily aligned with the direction of our inferences this is not always so. For example, though belief in a disjunction $p \vee q$ usually derives from belief either in p or in q, the order of evidence sometimes proceeds in reverse from $p \vee q$ to weak confirmation of p and q, as when, e.g. I infer from the mess in the corner that one of our two puppies has misbehaved again even though I can't tell which one. In such an instance we would maintain that $p \vee q$ is the case either because of p, or of q, or of both, even while remaining uncertain as to which explanation for $p \vee q$ is the correct one. Thus even if the concept of "because" has its psychological genesis in the experience of credibility derivation, what we mean by saying 'p because q' is something other than that belief in q is our reason for believing p. (Just the same, I shall contend that important if obscure analytic ties still remain between the structure of inference and the structure of responsibility despite the lack of simple equivalence between these.[9])

Now consider a typical problem in the theory of explanation. Suppose that

(20) Dale Smith was sired by Adam Smith,

(21) Dale Smith is a male,

and that moreover,

(22) All of Adam Smith's children are males.

Can or can we not explain (21) by appeal to datum (20) and generalization (22)? Clearly the proffered explanation fits the Hempel-Oppenheim "covering law" model of explanation; yet as many writers have protested, one might just as well argue instead that given (20), (21) is a necessary condition for the truth of (22) and, along with the sex particulars about Adam Smith's other children, is *why* generalization (22) is true. What is uncertain in this case is the direction of responsibility: Given that Adam Smith is Dale Smith's father, is Dale masculine *because* it is a law that all Adam's children are male, or are all of Adam's children male simply because Dale, like his two siblings, happened to turn out male? Either alternative is possible, for while we have reason to think that about 13% of three-children families should have all boys by chance alone, it is also plausible that aberrant gamete formation in some men prevents them from siring offspring of both sexes. Either way—and this is the point of this deliberately ambiguous example—it is evident that (20) and (22) are an intuitively acceptable "explanation" of (21) if and only if the latter is considered to be *due* to the former.

[9]A similar situation obtains for many of the important problematic concepts around which we order our lives. For example, the notion of propositional probability almost certainly originates in the primitive use of declarative assertions of form 'Probably p' to communicate less-than-perfect confidence in the truth of proposition p. Yet what is meant by "the probability of p" in more sophisticated contexts today is clearly distinct from any claim about the degree to which any person does, in fact, believe p. It does, however, retain an intimate if still controversial connection with the degree to which p *should* be believed.

Although 'p because q' entails both p and q, the subjunctive 'Were q to be the case, then, due to q, p would also be the case' preserves the responsibility attribution of the former while remaining noncommittal about the truth of p. And this, I suggest, is essentially the force of the non-truth-functional 'If ... , then ...'. When I fear, e.g., that my car won't start tomorrow morning if it freezes tonight, but do not consider the state of affairs so conjectured to matter for whether it *will* freeze tonight, my belief is that a freeze tonight would *bring about*—i.e. be responsible for—an ignition failure in my car, but that the state of affairs which is the basis of this contingency has no influence on the weather. (Otherwise, if I feared, say, merely some state of affairs s in light of which the hypothesis of its freezing tonight would be a good *reason* for inferring that my car won't start tomorrow, the question would remain whether s is anything more than the disjunction of its not freezing and my car's not starting, and why I should think that the conditional probability of a freeze, given s, is the same as its prior probability. My further supposition about the order of responsibility is needed to *justify* my construing the probability of a freeze as independent of s.) Similarly, when I surmise that all As are Bs in the sense of the connective under which primary induction yields a pattern such as (14), I hypothesize a connection between properties A and B which I have no reason to think affects the prevalence of A but which, once an instantiation of A has occurred, acts with the latter to produce an instantiation of B.

That ascriptions of responsibility are a fundamental and pervasive theme in our *de facto* beliefs about reality's organization admits of no real doubt.[10] How to do justice to this intuition in formalized reconstructions of natural language, on the other hand, is quite another matter. (One could, of course, argue that the notion is bereft of all validity and should hence be ignored, but a more seemly philosophic reaction is to analyze it first and worry about its epistemic status afterward.) Whereas past skirmishes with this problem along traditional lines have had an unblemished history of failure (witness the state-of-the-art regarding subjunctive and counterfactual conditionals), it now appears that our primary explicative access to propositional responsibility may well lie in the latter's relation to probability structure. The exact contours of this relation, however, are by no means simple to trace, for the key issues are intricately technical and quickly fuse with advanced problems on the nature of propositional probability. Since it is impossible to do even frontier justice to these issues without eclipsing the main concerns of this paper, I shall here attempt no more than to outline with minimal

[10]The reader who, like one referee of this paper, is sceptical of this claim is invited to test how long he can conduct his practical affairs without any recourse, explicit or otherwise, to the concept of "because" and its cognates. Even if one were to argue that there is no order of responsibility in the external world but only a "pragmatic" structure of inference wherein some beliefs are reasons for others, this would still concede that we hold some beliefs *because* of our reasons for them and hence admit an order of responsibility at least among cognitive events.

argument what seems to me to be the most promising position on the matter. Specifically, I propose that while 'because,' 'is due to,' and similar expressions function grammatically as propositional connectives, they are parasitical upon other, more basic connectives by way of the probabilistic implication patterns projected by the latter.

By an "n-adic propositional connective," let us mean any n-place propositional matrix F such that (i) for any n-tuple $\langle p_1, \ldots, p_n \rangle$ of propositions, $F(p_1, \ldots, p_n)$ is also a proposition, and (ii) matrix F contains no intact propositions nor any intentional-act verbs such as 'believes', 'hopes,' 'perceives,' etc. We can differentiate between "truth-functional" and "modalic" propositional connectives as follows: Let a "truth state" of proposition n-tuple $\langle p_1, \ldots, p_n \rangle$ be defined as any conjunction of n propositions $p^*{}_1, p^*{}_2, \ldots, p^*{}_n$ such that each $p^*{}_i$ is either p_i or $\sim p_i$. Then,

Definition 1. An n-adic propositional connective F is *truth-functional* iff, for any $n+3$ propositions $p_1, \ldots, p_n,\ b,\ t_i$ and t_j such that t_i and t_j are truth states of $\langle p_i, \ldots, p_n \rangle$ while neither $\Pr[t_i \cdot F(p_1, \ldots, p_n) \cdot b]$ nor $\Pr[t_j, \cdot F(p_1, \ldots, p_n) \cdot b]$ are zero,

$$\frac{\Pr[t_i \mid F(p_1, \ldots, p_n) \cdot b]}{\Pr[t_j \mid F(p_1, \ldots, p_n) \cdot b]} = \frac{\Pr(t_i \mid b)}{\Pr(t_j \mid b)}.$$

That is, F is truth-functional iff, for any argument n-tuple $\langle p_1, \ldots, p_n \rangle$ and relative to any background information b, $F(p_1, \ldots, p_n)$ does not affect the likelihood ratios of those truth-states of $\langle p_1, \ldots, p_n \rangle$ with which $F(p_1, \ldots, p_n)$ is compatible. It is easily seen that any connective which can be defined via truth table is truth functional in the present sense. (More precisely, it can be proved that a connective is truth functional if and, so long as it is compatible with at least two distinct truth states of its argument, only if $\Pr[F(p_i, \ldots, p_n) \mid t] = 1$ for every truth state t of $\langle p_1, \ldots, p_n \rangle$ such that $\Pr[(F(p_1, \ldots, p_n) \cdot t)] > 0$.) In contrast,

Definition 2. A propositional connective is *modalic* iff it is not truth-functional.

The justification for introducing this second category of propositional connectives is simply that, as we have seen, the connective 'If ..., then ...' is in practice often construed modalically.

In principle, we should be able to distinguish many different types of modalic connectives according to the various patterns of probability they confer upon their arguments' truth-states. Which of such patterns are characteristic of connectives actually in use is still largely unknown to me; the best I can do on this occasion is to describe two extremes which seem to idealize probabilistic inference patterns to which our thinking does, in fact, often conform.

Definition 3. A dyadic propositional connective $\Rightarrow$ is a *modalic entailment* iff for any two propositions p and q for which the following probabilities are well-defined, (a) $\Pr(p \mid p \Rightarrow q) = \Pr(p)$, (b) $\Pr[q \mid (p \Rightarrow q)] = 1$, and (c) $\Pr[q \mid (p \Rightarrow q) \cdot \sim p] = \Pr(q \mid \sim p)$.

Definition 4. A dyadic propositional connective $\Rightarrow$ is a *modalic determination* iff, for any two propositions p and q for which the following probabilities are well-defined, (a) $\Pr(p \mid p \Rightarrow q) = \Pr(p)$, (b) $\Pr[q \mid (p \Rightarrow q)] = 1$, and (c) $\Pr[q \mid (p \Rightarrow q) \cdot \sim p] = 0$.

The difference between Definitions 3 and 4 is that whereas p is only a sufficient condition for q, given $p \Rightarrow q$, when $\Rightarrow$ is a modalic entailment, it is both necessary and sufficient when $\Rightarrow$ is a modalic determination.

It should be observed that modalic entailment and determination are here defined only in terms of their *primary* inductive implications. Nothing is said about probabilities contingent upon $p \Rightarrow q$ when this is supplemented by additional background information b. How $\Pr[p \mid (p \Rightarrow q) \cdot b]$ and $\Pr[q \mid (p \Rightarrow q) \cdot \sim p \cdot b]$ may be affected by secondary induction from b remains an open question. Neither shall I here attempt to identify any patterns of weak modalic implication wherein $\Pr[q \mid (p \Rightarrow q) \cdot p]$ is less than unity, even though some such notion appears needed to formalize our commonsense belief in fallible conditionality—as when, e.g. I warn "If you don't slow down, you'll get a speeding ticket" even though I know that your getting ticketed is not *certain* to follow from your continued speeding.

Insomuch as modalic entailment and modalic determination are *categories* of propositional connectives, definition of the type does little to make known what instances, if any, fall under it. While commonsense is adamant that modalic entailments and determinations (or at least approximations thereto) do exist, it has been extremely frugal with specific examples—in fact, about the only one I can think of is causation, though to be sure it is an open question whether the unreconstructed everyday use of "cause" may not sprawl across what more refined analysis will eventually recognize as a whole family of modalic connectives. On the other hand, existential use of a category does not require familiarity with its members, and we are perfectly free to introduce derivative concepts of modalic entailment and determination which apply directly to propositions rather than to propositional connectives as follows:

Definition 5. Proposition p modalically entails (determines) proposition q iff there exists, at least in linguistic potential, a propositional connective $\Rightarrow$ such that $p \Rightarrow q$ while $\Rightarrow$ is a modalic entailment (determination).

(By saying that a connective exists "in linguistic potential," I mean that it could be added without ontological error to our language even if we do not in fact currently recognize it. The clause about "linguistic potential" can be omitted if propositional connectives are regarded not as *de facto* linguistic entities but as propositional relations whose reality is independent of their recognition by any extant language.)

It may seem intolerably irresponsible to introduce a connective such as this whose valid utilization requires the existence of still other connectives for which our only evidence lies buried in primitive intuition and of whose nature we have only a feeble conception. Yet this, I contend, is just what in effect we do in our ordinary-language ascriptions of propositional responsibility. For when construing 'If p, then q' to make no difference for the credibility of p, we seldom if ever intend any judgment about what particular connection between p and q justifies this inference; in order for what we mean by this statement to be true, it quite suffices that $\Pr(p \mid \text{If } p \text{ then } q)$ *be* equal to $\Pr(p)$. I suggest, therefore, that when 'If p, then q,' is asserted as a genuine conditional, it is usually equivalent in force to 'p modalically entails q'. Similarly, 'q because p' may as a first approximation be equated with 'p modalically entails q, and p is the case.'[11]

VI

The preceding sections of this paper have unearthed shards of a profoundly important but heretofore neglected dimension of human reason. These fragments are, admittedly, too small to reveal a very coherent picture, but rather than continuing to dig for more of the missing pieces—which would only break open a viper's nest of further perplexities—I shall try in closing to put the fundamental issue into clearer perspective by resurveying it from a slightly different vantage point.

While I have proposed that the probabilistic implications projected by an assertion 'If p, then q' clarify whether the conditional is to be understood as 'q because p (unless not-p)', I do *not* claim that 'q because p (unless not-p)' is true (when it is) *because* its probabilistic implications are what they are. Rather, the latter are merely a symptom that p is a source of q. We cannot construe the responsibility ordering thus to be *due* to the probability patterning for, inter alia, the important

[11] I say "as a first approximation" for two reasons. One is that in practice we often accept 'q because p' even when we believe the probability of q, given p and our grounds for 'q because p' be less than unity. Secondly, and more importantly, it is not at all clear whether responsibility order in deductive systems—e.g., $p \vee q$ because p—can adequately be analyzed in terms of modalic entailment or its cognates. (I believe that it can, but the argument requires an undoubtedly controversial interpretation of propositional probability under which, e.g., the probability of a tautologically true proposition t is less than unity when we lack sufficient information to tell that t is tautological.)

reason that in order to derive the latter we must first presuppose a responsibility structure for the probabilities themselves.

Consider, for example, the system of credibilities generated by two propositions p and q, namely, the set of all probabilities $\Pr(t_i \,|\, t_j)$ such that propositions t_i and t_j are both truth functions (with t_j logically consistent) of exactly p and q. Although there are $(2^4)(24^4 - 1) = 240$ of these probabilities, they have only three degrees of freedom in that the numerical values of any three which are mathematically independent of one another suffice to determine the numerical values of the remainder. For example, $\langle \Pr(p), \Pr(q \mid p), \Pr(q \mid \sim p) \rangle$, $\langle \Pr(q), \Pr(p \mid q), \Pr(p \mid \sim q) \rangle$, and $\langle \Pr(p \cdot q), \Pr(p \cdot \sim q), \Pr(\sim p \cdot q) \rangle$ are three such alternative bases for the system generated by p and q. But which probabilities in the system are the sources of the others—i.e., which ones *account for* why the rest have the values they do have ? In particular, insomuch as

$$\begin{aligned}
\Pr(p \cdot q) \quad &= \Pr(p) \times \Pr(q \mid p) = \Pr(q) \times \Pr(p \mid q), \\
\Pr(p \cdot \sim q) \quad &= \Pr(p) \times [1 - \Pr(q \mid p)] = [1 - \Pr(q)] \times \Pr(p \mid \sim q), \\
\Pr(\sim p \ \cdot q) \quad &= [1 - \Pr(p)] \times \Pr(q \mid \sim p) = \Pr(q) \times [1 - \Pr(p \mid q)], \\
\Pr(p) \quad &= \Pr(p \cdot q) + \Pr(p \cdot \sim q), \\
\Pr(q) \quad &= \Pr(p \cdot q) + \Pr(\sim p \ \cdot \ q),
\end{aligned}$$

should we say that $\Pr(p)$, $\Pr(q \mid p)$,and $\Pr(q \mid \sim p)$ are jointly responsible for $\Pr(p \cdot q)$, $\Pr(p \cdot \sim q)$, $\Pr(\sim p \ \cdot q)$, and $\Pr(q)$; that $\Pr(q)$, $\Pr(p \mid q)$, and $\Pr(p \mid \sim q)$ are jointly responsible for $\Pr(p \cdot q)$, $\Pr(p \cdot \sim q)$, $\Pr(\sim p \cdot q)$ and $\Pr(p)$; that $\Pr(p \cdot q)$, $\Pr(p \cdot \ \sim q)$ and $\Pr(\sim p \ \cdot q)$ are jointly responsible for $\Pr(p)$ and $\Pr(q)$; or that these probabilities have still some other responsibility structure? Only when we decide which of these probabilities are the origin of the remainder can we begin to determine what their numerical values are.

"But," you protest, "why is any notion of responsibility needed here? Why can't I derive the numerical values of this probability system from whatever arbitrary basis for it I choose?" Fair enough–if you can do it thus arbitrarily. But in order for you to think your way to any consistent set of interdependent numerical probabilities, you must ground your conclusions about some of these upon your preceding judgments about others, and your choice of a pattern of reasoning is precisely wherein lie your presuppositions about the structure of propositional responsibility. Suppose, for example, that you are trying to decide the credibility of proposition p. Insomuch as $\Pr(p) = \Pr(p \cdot q) + \Pr(p \cdot \ \sim q)$ for any other proposition q, would or would it not be rational for you to reach a judgment about the numerical value of $\Pr(p)$ before you judge $\Pr(p \cdot q)$ and $\Pr(p \cdot \sim q)$? (Note that the question is *not* whether you could ever obtain evidence e which allows you to judge $\Pr(p \mid e)$ before $\Pr(p \cdot q \mid e)$ and $\Pr(p \cdot \sim q \mid e)$. That would only raise the prior problem of whether, insomuch as $\Pr(p \mid e) = \Pr(p \cdot e)/\Pr(e) = \Pr(p \cdot e)/[\Pr(p \cdot e) + \Pr(\sim p \cdot e)]$, you do not have to decide about $\Pr(p \cdot e)$ and $\Pr({\sim}p \ \cdot \ e)$ before you can determine $\Pr(p \mid e)$.) It is simple to show by argument from vicious regress that if rational judgment of probabilities is to be possible at all, it cannot be the case that for every two distinct propositions p and q, determination of $\Pr(p \ \cdot \ q)$ and $\Pr(p \cdot \ {\sim} \ q)$ is prerequisite to determining $\Pr(p)$. In those instances in which $\Pr(p)$ *can* properly be judged without prior judgment of p's joint probabilities with q and ${\sim}q$, let us say that p is *credibilistically independent* of q, or "CrInd(p, q)" for short. If both CrInd(p, q) and CrInd(q, p) presumably $\Pr(p \ \cdot \ q) = \Pr(p) \times \Pr(q)$ where P(p) and Pr(q) are decidable independently of one another. (This is not a theorem of the probability calculus; rather, I propose that it is a fundamental axiom governing rational judgment.) If CrInd(q, p) but not CrInd(p, q) the other hand, your judgment of $\Pr(p)$ must derive from your judgments of $\Pr(p \ \cdot \ q)$ and $\Pr(p \cdot \ {\sim} \ q)$, while to arrive at the latter following an independent determination of $\Pr(q)$ you first need $\Pr(p \mid q)$ and $\Pr(p \mid{\sim} q)$. But to put $\Pr(q)$, $\Pr(p \mid q)$, and $\Pr(p \mid \sim q)$ conceptually prior to $\Pr(p)$ is tantamount to holding that the latter is what it is *because* the former are what they are, and if you further judge that $\Pr(p \mid q) \neq \Pr(p \mid \sim q)$ you are in effect maintaining that whether or not q is the case is, at least in part, a *source* of whether or not p is the case. Or at least this is so for me: While I am still profoundly ignorant of what, specifically, must be true of two propositions in order that one be credibilistically independent of the other, my linguistic intuition is adamant that Crlnd(p, q) is false if p's being the case would in any way be due to q. In particular, it is introspectively evident to me that the reason I would never consider it rational to judge the probability of a truth-functional molecular proposition $F(p_1, \ldots, p_n)$ before determining the probabilities of the various truth states of $\langle p_1, \ldots, p_n \rangle$ is precisely that I consider the latter to be the source of $F(p_i, \ldots, p_n)$'s truth value. I submit, then, that until such time as it can be shown that our numerical conclusions about a system of prior probabilities are independent of the pattern of argument by which we arrive at them, a choice of one such pattern rather than another is *justified* only if we consider this to reflect the order of determination among the propositions at issue. If so, it is an inescapable correlate of our having any rational basis for uncertain belief at all that we also be committed to accept a structure of propositional responsibilities.

Now consider the problem of whether natural regularities—i.e., "laws"—are a consequence of the particular instances which they subsume, or are instead a source of the latter. To work with the simplest possible example, let h be the hypothesis that all objects have a certain property Q, while $\langle a_1, \ a_2, \ \ldots \rangle$ is a set of entities which must possess Q if h is to be the case. Assuming that $(x)Qx$ is not analytically true, which propositions in the set $\langle h, \ Q(a_1), \ Q(a_2), \ldots \rangle$ are credibilistically independent of the rest? If the truth of h derives from that of the $Q(a_i)$, then on grounds of symmetry we have $\mathrm{CrInd}[Q(a_i), Q(a_j)]$ for each

$a_1 \neq a_j$, from which it follows by the fundamental axiom assumed above that $\Pr[Q(a_i) \cdot Q((a_j)] = \Pr[Q(a_i)] \times \Pr[(Q(a_j)]$ and hence that $\Pr[(Q(a_j) \mid Q(a_i)] = \Pr[(Q(a_j)]$. That is, if h is the case (if it is) *because* its instances $Q(a_1), Q(a_2), \ldots$ are all true, then observing that one object a_i has property Q does not alter the probability that Q also holds for another object a_j. On the other hand, if h is *why* its instances obtain, then observation of $Q(a_i)$ is able to confirm $Q(a_j)$ through its confirmation of h. Specifically, if responsibility goes from the $Q(a_i)$ to h, then $\Pr(h)$ can be no greater than the product of all the $\Pr[Q(a_i)]$ and approaches zero as the number of objects a_i grows indefinitely large;[12] whereas if we interpret h as prior to the individual $Q(a_i)$ in responsibility, then we can arrive at whatever non-zero value for Pr(h) seems appropriate and deduce as a consequence that $\Pr[Q(a_j) \mid Q(a_i)] > \Pr[Q(a_j)]$.[13] (Even $\Pr(h) = 0$ is compatible with h's being a source regularity through which $Q(a_i)$ confirms $Q(a_j)$, but this case is more technically advanced and need not be discussed here.) While the present example is far too simplistic to be more than suggestive about the responsibility ordering of those data and generalities which concern us in real life, the evidence is strongly presumptive that we cannot properly learn from experience—i.e., our knowledge of particulars already observed can make no rational difference for our expectations about the properties of objects yet unencountered—unless we admit of natural regularities which are at least in part the *sources* of particular events.

Because I do, in fact, perforce believe that properties which I have observed to occur in the past are the ones most likely to occur under similar circumstances in the future, I am thus compelled to accept that some natural regularities exist which *bring about* the features of the events they subsume, rather than resulting from them. But by what conceptual machinery is it possible to express such a regularity? *Truth-functionally defined generalizations will not do*; for in order of responsibility, truth functions are dependent upon their components rather than antecedent to them. For example, if $(x)Qx$ is construed to be the limiting case of a

[12] It is an inevitable extension of the fundamental CrInd axiom that if all propositions in a set $\langle p_1, \ldots, p_n \rangle$ are credibilistically independent of one another, then $\Pr(p_1 \cdot p_2 \cdot \ldots \cdot p_n) = \prod_{i=1}^{n} \Pr(p_i)$.

[13] Another way to make this point in very general terms is as follows : Let c_i and c_j be two conclusions which are entailed by hypothesis h, and assume that if it were not for h, c_i and c_j would be credibilistically independent of each other—i.e., that $\Pr(c_i \cdot c_j \mid \sim h) = \Pr(c_i \mid \sim h) \times \Pr(c_j \mid \sim h)$. Then it can be shown that

$$\frac{\Pr(c_j \mid c_i)}{\Pr(c_j)} = 1 + \frac{\Pr(h) \times \Pr(\sim h) \times \Pr(\sim c_i \mid \sim h) \times \Pr(\sim c_j \mid \sim h)}{\Pr(c_i) \times \Pr(c_j)},$$

which says that c_i confirms c_j so long as $0 < \Pr(h) < 1$ and neither c_i or c_j are certain given $\sim h$. The crucial assumption here is that c_i and c_j are inferentially unrelated if h is not the case. This is reasonable as a simplifying approximation if, were h to be the case, c_i and c_j would obtain *because* of h, but would not be acceptable if h *derives* from c_i and c_j, as when, e.g., h is defined as the conjunction of c_i and c_j. (This theorem remains unaltered if all the probabilities involved are replaced with probability densities, so a nonzero probability for h is not strictly necessary here.)

conjunction of propositions of form $Q(a_i)$, then $(x)Qx$ *derives* its truth value from that of its instances and cannot sustain the conjecture "$Q(a_i)$ because $(x)Qx$" nor provide grounds on which observations $Q(a_1), \ldots, Q(a_n)$ rationally increase the probability of $Q(a_{n+1})$. In order to assert that natural events are governed by *source* regularities, we require a distinctive array of quasi-logical concepts—call them "modalic operators"—whose grammatical behavior is more or less *like* that of the logical terms from which we construct statements of instance-derivative regularities, yet which will form generalities that are credibilistically independent of their instances. Just what specific modalic operators may occur in natural language or are needed by technical science and philosophy is very much an open question. Almost surely we must recognize one or more propositional connectives in the "modalic entailment" category, but do we require modalic quantifiers as well? (It would appear so if, e.g., generalities of form 'Everything has property Q' can be source statements when predicate 'Q' does not contain modalic operators.) For that matter, are the connectives which link predicates in generalized source conditionals truly *propositional* connectives, or may they not instead be higher-level relations whose arguments are irreducibly properties? (For example, if 'All As are Bs' is nothing more than a conditional-distribution assertion, formalizing it as $(x)(Ax \Rightarrow Bx)$—which entails '$Aa \Rightarrow Ba$ for any particular object a—rather than as '$A \Rightarrow B$' may be unsound unless '$pr(B \mid A) = r$ for $r < 1$' is similarly read as $(x)(Ax \overset{r}{\Rightarrow} Bx)$ in which '$\overset{r}{\Rightarrow}$' denotes a propositional connective akin to but weaker than modalic entailment. Actually, when 'All As are Bs' has probabilistic implications [11], it is *not* saying merely that $pr(B \mid A) = 1$,[14] but that there are important, difficult problems here should be evident.) And will the same modalic operators needed to express basic laws also suffice for asserting generalities which, though responsible for their instances, are derivative from more fundamental laws, or do we need a hierarchy of modalic concepts corresponding to a responsibility hierarchy or source regularities? These are just a few of the puzzles which bob in the wake of recognition that truth-functional generalizations alone cannot support our conception of a lawful universe, and their exploration should provide many happy hours of philosophical bemusement for decades to come.

EPILOG

The intent of this paper has been to incite concern for two hitherto overlooked

[14]Specifically, $pr(B \mid A) = 1$ (or any other probability of property B given property A) is in itself neutral with respect to whether B is due to A, non-A is due to non-B, or neither of these. As a result, interpreting "All As are Bs" to have implications (11) presumes it to assert not merely that $pr(B \mid A) = 1$ but also that A brings about B. More generally, the fact that primary induction construes conditional property-probabilities to carry no information about the incidence of the reference class reveals that primary induction begins with a supposition about the direction of responsibility.

themes in the theory of inference which provide important new leverage upon many currently unsolved problems in the philosophy of science; and if its emphasis on provocation rather than finesse has led to grievous errors of detail, these may at least inspire others to domesticate this wilderness by setting them aright. The themes in question are (1) our unavoidable belief in a responsibility, derivational, or "becausal" ordering of propositions, and (2) the possibility of diagnosing a proposition's meaning through consideration of its probabilistic implications. Regarding (1), I have intimated above that many perplexities of "explanation" reduce to questions about the direction of propositional responsibility, and while I shall not develop the thought here, there is also reason to hope that suitable recognition of responsibility seniorities will do much to ameliorate the distress of Goodman's "new riddle of induction" (i.e., the "green/grue" clash in inductive extrapolations), as well as other philosophic *contretemps* in which an intuitively plausible form of argument (e.g., the "principle of insufficient reason" for determining prior probabilities) yields inconsistent conclusions when applied to alternative linguistic partitionings of the same underlying set of ideas. And as for (2), we have seen from analysis of Hempel's "paradox of confirmation" that two prima facie equivalent statements may project different patterns of inductive implications and must accordingly be suspected to assert different propositions. In particular, the probabilistic inferences drawn both in everyday life and in technical science from assertions of conditionality show that unless natural reason is systematically fallacious in this respect, the most basic sense of 'If p, then q' cannot be defined truth-functionally and is not, in this "modalic" sense of the conditional, equivalent to 'If not-q, then not-p' even though both have the truth-functional $\sim (p \cdot \sim q)$ as a consequence. More generally, non-truth-functional connectives such as 'is a cause of' are essential for expressing the laws of a world which is truly lawful, even if only probabilistically so; and while this paper has made no attempt to clarify the semantic or epistemological status of such modalic concepts, it may be conjectured that if attempts at explicit definition fail, we can at least defend them as theoretical terms introduced by axioms which stipulate, inter alia, that they support probabilistic conclusions in accord with the patterns by which we do, in fact, interpret them.

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